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Endogenous Economic Fluctuations

Studies in the Theory
of Rational Beliefs

With 4 Figures
and 19 Tables

Chapter 9: Asset Prices With

Rational Beliefs

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9. Asset prices with rational beliefs*

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Summary. This paper introduces the concept of Rational Belief Equilibrium (RBE) as a basis for a new theory of asset pricing. In an RBE the beliefs of agents are, in general, wrong in the sense that they are different from the true probability of the equilibrium process. These beliefs are, however, rational. Consequently, in an RBE agents make forecasting mistakes and these play a crucial role in the analysis. First, these mistakes are the reason why stock returns are explainable. Second, the aggregation of these mistakes generates Endogenous Uncertainty: it is that component of the variability of stock prices which is endogenously propagated. We develop some propositions and empirical implications of the theory of RBE to asset pricing. Based on data for the post world war II era, we formulate an econometric model of stock returns which allows non-stationarity in the form of changing regimes. A sequence of econometric hypotheses are then formulated as implications of the theory of RBE and tested. The empirical analysis shows that

(i) common stock returns are forecastable within each environment but it takes time for agents to learn and approximate the forecasting functions. For some agents the time is too short so that it is too late to profit from such learning;
(ii) the equilibrium forecasting functions change from one environment to the other in an unforecastable manner so that learning the parameters of one environment does not improve the ability to forecast in the subsequent environments.
(iii) more than 2/3 of the variability of stock returns is due to endogenous uncertainty.

JEL Classification Numbers: D84, G12, G14.

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Introduction

What should be the conceptualization of an equilibrium valuation of assets in the absence of markets for contingent claims? In this paper we propose a new equilibrium concept for the pricing of securities in a multi-period economy. The central distinguishing characteristic of our approach is the use of the theory of "Rational Beliefs," developed in our earlier papers (Kurz [1994a], [1994b]) and is reviewed in the *Editor's Introduction* to this volume. In order to define the concept of a *Rational Belief Equilibrium* (RBE) (see Kurz [1994b]), we construct a simple general equilibrium model with financial securities and heterogeneous agents. The model is simple because our aims are narrowly restricted to an explanation of the essential properties of an RBE and of the empirical implications of these properties. The model can be generalized without changing the basic properties discussed here and this fact is the basis for our empirical work which explores the econometric implications of the theory of RBE to asset markets. We do not aim to investigate in this paper analytical questions such as the consistent construction of a price state space and the existence of RBE. The reader may consult other papers in this volume which address these issues (see Henrotte [1996], Kurz and Schneider [1996], Kurz and Wu [1996] and Nielsen [1996]).

The central feature of an RBE originates in the assumption that the agents do not possess structural knowledge. That is, they do not know the demand functions of other agents and therefore they cannot compute general equilibrium prices. Also, they do not know the true probability of the random variables in the economy. As in Kurz [1994a], agents know only the stationary measure of the equilibrium process and "rationality" of beliefs is defined by the requirement that the beliefs be compatible with the stationary measure. In an RBE all agents hold Rational Beliefs with respect to the true equilibrium probability of the observed variables.

In a typical RBE the uncertainty of agents about future prices is treated in the same way as their uncertainty about any other random variable in the economy. Consequently, an RBE has the crucial property that the variability of prices is not necessarily tied to the variability of the "fundamental" exogenous variables since it also depends upon the "state of belief". This leads to the emergence of "endogenous uncertainty", a concept introduced in Kurz [1974]. Apart from the objective of showing the applicability of the theory of RBE to asset pricing, we aim to demonstrate that endogenous uncertainty is both a useful analytical concept as well as an empirically viable one.

We start by examining in the first section below an episode of price movement on the New York Stock Exchange. We call this episode "*The Puzzle of the Declining Stock Prices*". After developing the concept of an RBE with securities and exploring its central properties, we shall offer extensive econometric testing of their implications. At the end we return to the Puzzle and demonstrate how to use the machinery of RBE to explain the observed data.

1 The puzzle of the declining stock prices

Figure 1 presents a graph of the nominal value of the Dow-Jones index of 30 stocks on the New York Stock Exchange during the post war period. Since the Dow-Jones
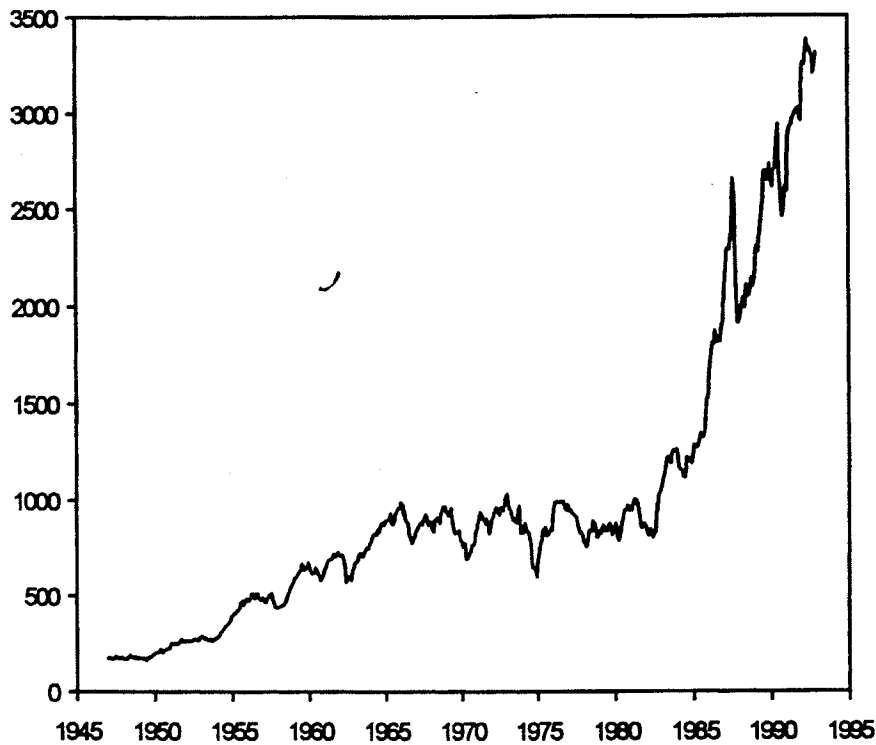
Figure 1

Table 1

	GNP (billions, current dollars)	Corporate Profits (billions, current dollars)	Dow-Jones (January)	GNP Deflator	CPI
1965	702.7	82.9	889.9	96.4	97.2
1966	769.8	88.6	985.9	100.0	100.0
1983	3,405.0	212.7	1,064.3	296.9	307.4
1984	3,777.2	264.2	1,258.9	307.7	320.7

Source: Economic Report of the President, 1992, Tables B.1, B.3, B.56 and B.85.

is the market value of a basket of securities, the term "nominal" means that the dollar value of this basket has not been deflated. The graph reveals an extraordinary phenomenon: in 1966 the Dow-Jones reached the level of 1,000 but did not close above it. During the next 17 years of 1966–1983 the nominal index fluctuated in the wide band between 600 and 1,000. Other nominal indexes give a similar picture. To see why we think that this is extraordinary consider the nominal value of GNP and corporate profits in the U.S. during this period. Table 1 below shows that during this

Table 2. Dow-Jones/Consumer Price Index (selected years. CPI with 1966 = 100)

January, 1966	977.15
January, 1971	703.31
January, 1975	402.01
January, 1982	288.38
January, 1983	346.79
January, 1992	682.93

period the general price level in the economy rose by about a factor of 3, the nominal GNP rose by more than a factor of 3 while corporate profits, which were recovering from the 1981–82 recession, just kept up with inflation. Suppose now that the valuation of 1,000 in the Dow-Jones in 1966 is justified by a reasoning based on a rational expectations model according to which agents know the *true* stochastic law of motion of the economy. Such an equilibrium theory of asset prices is typically formulated in real terms and consequently has the zero homogeneity property with respect to commodity prices. Since the price level rose by a factor of 3 between 1966 and 1983 it would follow that all nominal values of assets should have increased by a factor of approximately 3 and that the Dow-Jones should have risen to about 3,000 in 1983. There was some difference in circumstances between 1966 and 1983 and allowance should be made for that. Yet, this should not change the general assessment. It is not easy to think of a reasonable rational expectations model that would explain both 1966 as well as 1983 and would not view the difference as a pure random error[1].

To see the dimension of real decline in stock prices during this period we compute in Table 2 the ratio of the Dow-Jones average to the CPI (1966 = 100). This information completes the central component of our puzzle: why did the prices of securities of the major U.S. corporations *decline in real terms by some 71%* between 1966 and 1982 while the U.S. economy continued to grow in real terms and aggregate corporate profits continued to rise so as to keep up with inflation?

Although the above episode is rather dramatic in nature its basic ingredients reappear, with varying intensities, at different times. This example is, therefore, representative of the range of phenomena which the theory of RBE addresses.

2 Rational belief equilibria of financial markets

2.1 A model of asset pricing

Consider an economy with K infinitely lived agents and a single consumable and perishable good whose price is set equal to 1 at all dates. The economy has only one

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[1] Grossman and Shiller [1981] utilize a rational expectations framework in examining the data from 1889 to 1979. They find the period under consideration to be substantially out-of-line and refer to it as a "breakdown of the model" (page 226) which they cannot explain.

productive activity (like Lucas's [1978] orchard) which generates an exogenous random stream $\{r_t, t = 1, 2, \ldots\}$ of returns ("dividends"). This activity is organized as a firm and at date $t = 0$ the distribution of ownership shares $\theta_0^k > 0$ among the agents is given with $\sum_{k=1}^{K} \theta_0^k = 1$. There exists a competitive market in which ownership shares are traded at a unit share price of P_t at date t. If agent k purchases θ_{t-1}^k shares at date $t - 1$ he is entitled to receive the dividends which are distributed at date t. The announcement of these dividends is made *before* trading resumes at date t. Hence at date t the ownership shares of agent k have a market value of $\theta_{t-1}^k(P_t + r_t)$. There are no transaction costs.

A second financial asset is a zero net supply, one period "bill" which is risk free. The owner of one unit of the bill at date t receives with certainty, at date $t + 1$, one unit of the consumption good. The price of a bill at t is q_t hence the risk free interest rate ρ_t satisfies $q_t = \dfrac{1}{1 + \rho_t}$. Denote by B_t^k the number of bills purchased at date t by agent k at the discount rate q_t.

Let c_τ^k be the consumption of agent k at date τ. Let his utility at t be the random variable

$$\sum_{\tau=t}^{\infty} \left(\frac{1}{1 + \delta} \right)^{\tau - t} u_k(c_\tau^k). \tag{1}$$

δ is common to all the agents. Now let W_t^k be the wealth of agent k at date t then by definition $W_t^k = \theta_{t-1}^k(P_t + r_t) + B_{t-1}^k$ and it is clear that a large negative r_t can cause problems of infeasibility. To avoid the possibility of bankruptcy and ensure the feasibility of a strictly positive endowment at all dates we assume that it is known by all the agents that there is an $\underline{r}$ such that

$$r_t \geqq \underline{r} > 0 \quad \text{with probability 1.} \tag{2}$$

(2) is a customary assumption in the finance literature. Indeed, it follows from (2) that there exists a lower bound $\underline{P}$ such that at all t $P_t \geqq \underline{r} \sum_{j=0}^{\infty} q_t q_{t+1} \cdots q_{t+j} = \underline{P}$ and the sum on the right hand side of the inequality must be finite with probability 1 if an equilibrium is to exist. Consider the collection of real numbers defined by all realizations of $\sum_{j=0}^{\infty} q_t q_{t+1} \cdots q_{t+j}$. Let λ be the infimum over this set and typically it is a number bigger than 1. This implies that with probability 1, $P_t + r_t \geqq (1 + \lambda)\underline{r}$. One way to ensure that $W_t^k > 0$ with probability 1 is for the agent to be required to select at any date a pair (θ^k, B^k) such that (i) $\theta^k \geqq 0$, (ii) $B^k \geqq -(1 + \lambda)\theta^k \underline{r}$. With these conditions of ensuring the endowment positivity of agent k we describe his budget constraint at date t as

$$\left\{ (c_t^k, \theta_t^k, B_t^k) \,\middle|\, \begin{array}{l} W_t^k = \theta_{t-1}^k(P_t + r_t) + B_{t-1}^k \\ W_t^k = \theta_t^k P_t + c_t^k + B_t^k q_t \\ c_t^k \geqq 0, \ \theta_t^k \geqq 0, \ B_t^k \geqq -(1 + \lambda)\theta_t^k \underline{r} \end{array} \right\}. \tag{3}$$

Let $[\underline{r}, \infty] = V$ and let the feasible set X be a Borel subset of $\mathbb{R}^3$ where $(r_t, P_t, \rho_t) \in X$. The exogenous process $\{r_t, t = 1, 2, \ldots\}$ is assumed to be a *stable process* on the probability space $(V^\infty, \mathscr{B}(V^\infty), \Pi_r)$ where Π_r is a true and exogenously given probability. The symbol $\mathscr{B}$ designates in this paper the Borel σ-field of the relevant

space. The probability Π_r is not known by any agent. The object of uncertainty for the agents is the sequence $\{(r_t, P_t, \rho_t), t = 1, 2, \ldots\}$ of dividend payments of the firm, stock prices and risk free interest rates (we shall use the term "prices" to refer to the pair (P, ρ)). Agents do not know the true probabilities of the process of prices and dividends and their uncertainty is represented by a probability on the measurable space $(X^\infty, \mathscr{B}(X^\infty))$ which is the space of infinite sequences of the observables $\{(r_t, P_t, \rho_t), t = 1, 2, \ldots\}$. A *belief* of agent k is a probability Q^k on the space $(X^\infty, \mathscr{B}(X^\infty))$ such that the system $(X^\infty, \mathscr{B}(X^\infty), Q^k, T)$ is *a stable dynamical system*. T is the shift transformation.

To formulate the optimal decision functions of the agents we need to state the information structure. We use the common notation I_t to designate the state of information available at date t. The information of agents at the time of selecting their decision functions is then

$$I_t = ((r_1, P_1, \rho_1), (r_2, P_2, \rho_2), \ldots, (r_t, P_t, \rho_t)). \tag{4}$$

If Q^k denotes the probability belief of agent k, he uses it to calculate his expected utility at t

$$V_t^k = E_{Q^k}\left(\sum_{\tau=t}^{\infty} \left(\frac{1}{1+\delta} \right)^{\tau-t} u_k(c_\tau^k) \,|\, I_t \right).$$

We then start the process by setting $\theta_0^k > 0$, $B_0^k = 0$ and $\sum_{k=1}^{K} \theta_0^k = 1$. Hence at date 1 we have $W_1^k = \theta_0^k(P_1 + r_1) > 0$. Given Q^k, agent k mazimizes V_1^k by selecting an optimal sequence of decision functions $\{(c_t^k(I_t), \theta_t^k(I_t), B_t^k(I_t)), t = 1, 2, \ldots\}$ which depend upon information that will be available at future dates t. To complete the description of the state space we introduce the last two components. First the *"state of asset holdings"* at the start of date t, which is denoted by $(\theta_{t-1}, B_{t-1}) = ((\theta_{t-1}^1, B_{t-1}^1), (\theta_{t-1}^2, B_{t-1}^2), \ldots, (\theta_{t-1}^K, B_{t-1}^K))$. Second, the *"state of beliefs"* of the agents at date t $Q_t = (Q_t^1, Q_t^2, \ldots, Q_t^K)$ where Q_t^k is the conditional probability of agent k at date t. In equilibrium we must have the market clearing conditions

$$\sum_{k=1}^{K} \theta_t^k(I_t) = 1 \qquad t = 1, 2, \ldots \tag{5a}$$

$$\sum_{k=1}^{K} B_t^k(I_t) = 0 \qquad t = 1, 2, \ldots \tag{5b}$$

Since at date t the vectors $((P_1, \rho_1), (P_2, \rho_2), \ldots, (P_{t-1}, \rho_{t-1}))$ and $(r_1, r_2, \ldots, r_t)$ are known, equations (5a)–(5b) *define equilibrium prices* (P_t, ρ_t) *as a sequence of random variables which depend only on* (r_t, I_{t-1}). We can write these as

$$P_t = \varphi_t(r_t, I_{t-1}, \theta_{t-1}, B_{t-1}, Q_t) \qquad t = 1, 2, \ldots \tag{6a}$$

$$\rho_t = \phi_t(r_t, I_{t-1}, \theta_{t-1}, B_{t-1}, Q_t) \qquad t = 1, 2, \ldots \tag{6b}$$

In general, the functions (φ_t, ϕ_t) depend upon t (since the domain of I_t changes with t). In the important case when the agents believe that the variables follow a Markov process, they can be written as time independent functions. These price functions together with the probability Π_r induce a joint distribution of

$((r_1, P_1, \rho_1), \ldots, (r_t, P_t, \rho_t))$ for all t and hence they induce a *true equilibrium probability* Π on the space of infinite sequences $\{(r_t, P_t, \rho_t),\ t = 1, 2, \ldots\}$. To keep our terminology simple we shall then say that the beliefs $(Q^1, Q^2, \ldots, Q^K)$ induce Π on $(X^\infty, \mathscr{B}(X^\infty))$. We are now ready to introduce the central concept of a *Rational Belief Equilibrium*:

Definition 1: *A Rational Belief Equilibrium (RBE) is a stable system* $(X^\infty, \mathscr{B}(X^\infty), \Pi, T)$ *of dividends, stock prices and risk free interest rates, a set of probabilities* $(Q^1, Q^2, \ldots, Q^K)$ *on the measurable space* $(X^\infty, \mathscr{B}(X^\infty))$ *which induce* Π *and are rational beliefs with respect to* Π, *and a sequence of decision functions* $\{(\theta_t^k(I_t), c_t^k(I_t), B_t^k(I_t)),\ t = 1, 2, \ldots\}$ *such that for* Π *a.e. we have*

(i) *Individual Optimum:* $\{(\theta_t^k(I_t), c_t^k(I_t), B_t^k(I_t)),\ t = 1, 2, \ldots\}$ *are optimal for* k *given* Q^k;

(ii) *Market Clearing:* $\displaystyle\sum_{k=1}^{K} \theta_t^k(I_t) = 1, \quad t = 1, 2, \ldots$

$$\sum_{k=1}^{K} B_t^k(I_t) = 0, \quad t = 1, 2, \ldots \qquad \qquad ($$

Our main aim in this paper is to explore the implications of RBE *as a theory of asset markets*. Before proceeding with this examination we need to define the key concept of agent's "mistake".

In an RBE no one knows the true equilibrium probability Π: the expectations of the agents are self-fulfilling not in the sense that $Q^k = \Pi$ for all k but in the sense that *the economy generates the long term statistics which are correctly anticipated by all the agents*! This is represented by the stationary measure of the equilibrium dynamics (see Kurz [1994a] and the *Editors General Perspective* which reviews this term). It follows that whenever diversity of beliefs is universal, almost all agents are "wrong" at almost all dates. Agent k is wrong in that his forecasts about the future are incorrectly made under the probability Q^k rather than the true probability Π. The idea that *rational agents can be wrong is a central conception of the theory of Rational Beliefs* and plays a central role in its application to asset markets. We denote the date τ forecast at date t under Q by $x_\tau^{eQ}(I_t) = E_Q(x_\tau | I_t)$, and introduce

Definition 2: *The mistake measure of agent* k *is the map* $M^k: \mathscr{B}(X^\infty) \to \mathbb{R}$ *defined by*

$$M^k(A) = \Pi(A) - Q^k(A) \qquad A \in \mathscr{B}(X^\infty). \tag{7}$$

The date τ *forecast mistake of agent* k *under* Q^k *is then* $M_\tau^k(I_t) = x_\tau^{e\Pi}(I_t) - x_\tau^{eQ^k}(I_t)$.

The concept of a "mistake" must be contrasted with the common term of a forecast "error" of an agent which arises because *he does not know the future*: a date $t + 1$ forecast error under any measure Q is defined by $\varepsilon_{t+1}^Q(I_{t+1}) = x_{t+1} - x_{t+1}^{eQ}(I_t)$.

2.2 The implication of RBE for asset pricing

We review now the main implications of the theory of RBE to asset prices. Aiming to keep the exposition simple we note that since we are dealing with diverse probabilities we need to keep in mind that all conditional expectations entail relationships which are true *almost everywhere* with respect to the appropriate probabilities. To

avoid repeating throughout the paper statements like "Q^k a.e." we would assume these to be understood from the context. Whenever confusion can arise we shall specify the probabilities involved.

2.2.A The relation of asset prices to exogenous fundamentals

In an RBE the distribution of beliefs have a real impact on asset prices and this motivates the definition of the concept of *Endogenous Uncertainty* (see Kurz [1974], [1993], [1994b] and Kurz and Wu [1996]). It is that component of the variability of asset prices over time which is caused by variations in the state of beliefs of the agents rather than by variations in the state of the exogenously specified dividend process. The state of beliefs is defined by the vector of conditional probabilities of the agents at each date. Now, since a rational agent can make mistakes it follows that in an RBE the market as a whole can experience large fluctuations in the valuation of assets which are not related to the exogenous "fundamentals". That is, the vector of conditional probabilities $(Q_t^1, \ldots, Q_t^K)$ which induces the price functions (φ_t, ϕ_t) may generate a valuation of assets which is drastically different from the valuation which would prevail under rational expectations and which is determined entirely by the correct state of the "fundamental" dividend process. Excess volatility of commodity prices due to endogenous uncertainty was investigated in Kurz [1994b] where the model is rather specialized. In the case of asset prices, one dimension of endogenous uncertainty is the dual phenomena of "*local bubbles*" and "*price amplification*", issues which we discuss now.

Suppose q_t^1 is an equilibrium price of some asset. The usual definition of a price "bubble" (see, for example, Blanchard and Watson [1982], and Tirole [1985]) proposes an alternative price $q_t = q_t^1 + b_t$ where the random sequence $\{b_t, t = 1, 2, \ldots\}$ satisfies certain expectations conditions. The random variables b_t are not observable and the standard models of bubbles imply that the sequence b_t is unbounded with probability one[2].

Local bubbles are episodes of price rises (or declines) which are not accompanied by a corresponding change in fundamental conditions of profits, endowments, preferences etc. At the end of the episode the price returns to its initial value.

Definition 3: *Let the equilibrium dynamics of the economy be of finite memory. A positive local bubble between dates t and t + N is a sequence of price realizations P_τ, $t \leq \tau \leq t + N$ (N is the length of the bubble and assumed longer than the memory of the system) where*

(i) *Prices P_τ satisfy*

    (a) $P_\tau \geq P_t$ *for* $t \leq \tau < t + N$ *with strict inequality at some date τ,*
    (b) $P_{t+N} \leq P_t$,

(ii) $r_\tau = r_t$ *all* $t \leq \tau \leq t + N$.

In a *negative* local bubble all the inequalities are reversed.

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[2] Blanchard and Watson [1982] proposed a mechanism of causing bubbles to "burst" occasionally so that they are only unbounded in expectations. The "bursting" is done by an artificial and exogenous random device which produces an i.i.d. signal. When the "bad" signal is observed by all traders the sample path of the bubble takes the value 0 and the price returns to its fundamental value.

A local bubble arises when the dynamics of prices and fundamentals do not correspond in that prices rise and fall without the fundamental conditions changing. The prediction of local bubbles in an RBE is important both theoretically as well as for empirical testing of the theory. Theoretically it is in contrast with any version of a rational expectation equilibrium in which prices are functions of the state of the dividend process and local bubbles cannot occur. Empirically it is generally not possible to observe periods in which exogenous variables do not change. However, the theory of RBE also predicts that for two different time intervals in which the exogenous conditions of an RBE are the same, *the time pattern of equilibrium prices will not be the same*. This way of stating what the theory implies is exactly how one would proceed to formulate an econometric model to test it.

An episode of *price amplification* is a time interval $[t, t + N]$ over which prices and dividends move in the same direction but the variability of the price series is greater than would be justified by the variability of the dividends. Amplification is formally analyzed in Kurz [1994b] and is part of the definition of Endogenous Uncertainty given in Kurz and Wu [1996] but is not central to this paper. We note that the empirical evaluation of such price movements is rather complex (see Shiller [1981], LeRoy and Porter [1981] and the subsequent controversy related to these papers).

Our interest in local bubbles and amplification arises from the fact that in a typical RBE *a positive fraction of all dates will belong to intervals in which the market exhibits local bubbles or price amplification*. Thus such patterns are generic to an RBE and distinguishes it from a rational expectation equilibrium. Of course, conditions of rationality prevent us from empirically establishing ex-ante that the market is in a local bubble but typically some evidence becomes available to enable an ex-post explanation of the reasons why a local bubble developed to begin with. For example, agents whom we can call "optimists" may form a belief that an environment of higher profits is just ahead and this will cause the price to rise. If such expectations are mistaken then, with high likelihood, the realization of profits will be disappointing. Conditioning upon the disappointing profit realization agents revise both their profits and price forecasts and this induces a price decline.

The theory of RBE makes predictions about the volume of trading which are not developed in this paper. In an equilibrium with diverse beliefs, local bubbles and market turning points (see (2.2.C) below) are associated with variations in the volume of trading as agents change their price forecasts *conditional upon the realization of prices*. In fact, ex-post, we can identify local bubbles through the variations in the volume of trade which take place without any exogenous "news".

2.2.B Excess returns in financial markets

For simplicity we consider only equilibria in which all agents have interior solutions for the optimization (1)–(4). The first order conditions of $k = 1, \ldots, K$,

all $t \geq 0$ are

$$E_{Q^k}\left[\frac{u_k'(c_{t+1}^k)}{u_k'(c_t^k)} \mid I_t\right] = \left(\frac{1+\delta}{1+\rho_t}\right) \tag{8a}$$

$$E_{Q^k}\left[\frac{u_k'(c_{t+1}^k)}{u_k'(c_t^k)} \frac{P_{t+1} + r_{t+1}}{P_t} \mid I_t\right]\left(\frac{1}{1+\delta}\right) = 1. \tag{8b}$$

Iterating (8b) and using standard transversality conditions we have that for all $t \geq 0$ the optimal solutions $(c_t^k, \theta_t^k, B_t^k)$ must satisfy

$$P_t = \sum_{n=1}^{\infty}\left(\frac{1}{1+\delta}\right)^n E_{Q^k}\left[\frac{u_k'(c_{t+n}^k)}{u_k'(c_t^k)} r_{t+n} \mid I_t\right]. \tag{9}$$

If the random variables r_t are uniformly bounded then in an RBE P_t are bounded random variables.

When agents know Π and if we let $H_{t+1} = (P_{t+1} + r_{t+1}) - (1 + \rho_t)P_t$ be the one period holding returns then, under risk neutrality, the "market efficiency" theory requires $E_{\Pi}(H_{t+1} \mid I_t) = 0$. In the case of risk aversion the holding returns of k are $H_{t+1}^k = u_k'(c_{t+1}^k)[(P_{t+1} + r_{t+1}) - (1 + \rho_t)P_t]$. Under rational expectations agents know Π and we have the *no excess returns* condition

$$E_{\Pi}(H_{t+1}^k \mid I_t) = 0. \tag{10}$$

This follows from (8a) and (8b) above when $Q^k = \Pi$. In an RBE Π is not known, $Q^k \neq \Pi$ and $(c_t^k, \theta_t^k, B_t^k)$, $t = 1, 2, \ldots$ are selected so that the condition which holds is $E_{Q^k}(H_{t+1}^k \mid I_t) = 0$. Hence (10) will, in general, fail and the "no excess returns" condition will not hold. The implication is that $(c_t^k, \theta_t^k, B_t^k)$ are not the best decision functions for k and this leaves the financial markets with extensive deficits and surpluses. The presence of excess utility returns in the market stands in sharp contrast with the conclusion of the "market efficiency" theory (see 2.2.C). It is important, however, to see that in an RBE an agent who adopts the consumption and trading rules $\{(c_t^k(I_t), \theta_t^k(I_t), B_t^k(I_t)), t = 1, 2, \ldots\}$ knows that these rules may not be optimal relative to the true probability Π but since he does not know Π he cannot compute the better rules.

The empirical implications of these conclusions are far reaching. A non-stationary approach implies that in an RBE stock returns may be "predictable" in the statistical sense of the term. This means that despite the fact that at each t $E_{Q^k}(H_{t+1}^k \mid I_t) = 0$ for all k, stock excess returns may exhibit correlation with some past variables. That is, from the failure of (10) it follows that at each t there exists a function $\xi_{t+1}^k(I_t)$ which depends upon information available at t such that

$$E_{\Pi}(H_{t+1}^k \mid I_t) = \xi_{t+1}^k(I_t). \tag{11}$$

Agents who discover the nature of the relevant information on the right hand side of (11) can generate excess returns. Thus, the recent literature on the forecastability of returns (see for example Campbell [1987], Campbell and Shiller [1988b], Fama [1981], Fama and French [1988], Fama and Schwert [1977] and Poterba and Summers [1988]) is entirely compatible with our RBE valuation theory. Moreover, we postulate in Section (3) below that non-stationarity can be approximated by

a random sequence of time intervals such that *within each interval* all joint distributions are time invariant. We refer to such intervals as "environments" (or "regimes"). The conclusion regarding the availability of excess returns in an RBE then implies that market excess returns *can be explained ex-post in terms of this sequence of shifting environments*.

2.2.C Information efficiency and the market efficiency theory

Our conclusions in (2.2.A)–(2.2.B) stand in sharp contrast with common conclusions of the market efficiency theory. However, the presence of excess returns does not imply the *informational* inefficiency of an RBE. One usually defines the market efficiency theory to mean that the market "processes information efficiently". This is expressed formally (see Fama [1970]) by the fact that agents use conditional probabilities to select their optimal decisions and given the familiar orthogonality conditions of conditional probabilities the conclusion follows. In this sense agents in an RBE utilize information efficiently since they use conditional probabilities to derive optimal decisions. There is, however, a profound difference between processing information and forming an assessment *about its meaning*. The theory of Rational Beliefs admits rational agents with diverse beliefs who make incorrect assessments of economic situations despite having efficiently processed all information. On the other hand, the market efficiency theory makes the fundamental assumption that the agents know Π, the true equilibrium probability, and consequently all of them agree on the correct *interpretation* of any information. Since we reject this assumption we view the market efficiency theory as a flawed doctrine which crucially depends upon an unrealistic assumption.

As for capturing excess returns, our theory does not imply that there is a riskless way for agents to extract excess returns; it does imply that *this risk has nothing to do with the standard measures of risk in terms of variances or covariances of random variables*. To clarify this central point note that under the hypothesis that one knows the true distribution of a random variable one can then define "risk" in terms of the moments of the distribution as is customary in the analysis of stochastic dominance. Under our assumption that agents do not have structural knowledge they face the more complex uncertainty of adopting or developing a theory about the structure of the dynamics of asset returns and the standard measures of market risk are irrelevant to this uncertainty. It is our belief that this perceived uncertainty is the dominant factor in asset markets although it is not clear how to objectively measure the subjective risk of "selecting the wrong theory".

In a broader context, an important implication of the RBE theory of asset prices is its vision of what is the function of trading in the market for securities. Contrary to the perspective of REE, agents know that the market is vulnerable to large scale mistakes of valuation. Consequently, the central function of trading in the market is to give agents at each date the opportunity to alter their portfolio in view of their *diverse* assessments that the valuation of any one or of all securities have changed too much in one direction or another. Contrary to REE which cannot explain why equally informed people trade in the market at all, trading in an RBE is an on-going process in which agents adjust their portfolios given their assessment in relation to

the market valuation. This highlights the important implication that without any exogenous "news" and even when all traders are equally informed we should expect to observe a substantial amount of trading which will be induced *by the equilibrium realization of prices*. The same realization induces diverse expectations of future prices and therefore trading.

2.2.D Orthogonality conditions and stationarity reversion

The generality of our next observation goes far beyond the specific model at hand and will be extensively used in the discussion below. Let $\lambda > 0$ and consider two events

$$B_t = \{x \in X^\infty : x_t - E_m x_t \geq \lambda\} \tag{12}$$

$$D_{t+j} = \{x \in X^\infty : x \in B_t \text{ and } x_{t+j} < x_t\}. \tag{13}$$

B_t is the event that x_t exceeds its expected value under the stationary measure m by at least λ. D_{t+j} is the event that following the realization of B_t, x_{t+j} *declines* below x_t. Now consider the averages

$$\Pi_D(\lambda) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} \frac{\Pi(D_{t+j})}{\Pi(B_t)}, \qquad Q_D^k(\lambda) = \lim_{N \to \infty} \frac{1}{N} \sum_{j=1}^{N} \frac{Q^k(D_{t+j})}{Q^k(B_t)}. \tag{14}$$

The theory of Rational Beliefs holds that an RBE has the following *stationarity reversion* properties:

(A) If B_t occurs then every agent k with Rational Belief Q^k will believe that with Q^k probability 1 there will be a first date τ, $t < \tau < \infty$ such that $x_\tau \leq E_m(x_\tau)$.

(B) As λ increases, the larger are both average probabilities $\Pi_D(\lambda)$ and $Q_D^k(\lambda)$ for all k.

The first component of stationarity reversion says that when x_t is above (below) its stationary forecast all agents are certain that in finite time x_τ will fall (rise) to a level below (above) the stationary forecast. This conclusion is rather weak; many non-stable processes (such as random walk) satisfy it as well. The more fundamental conclusion is (B). It asserts that as the deviation of x_t from its stationary forecast increases, the *average* probability increases that future values of x_{t+j} will revert back to their stationary forecast. The average probability increases for both the true equilibrium probability Π as well as the beliefs of all the agents. This conclusion is the heart of the stationarity reversion property of an RBE in that it establishes an asymmetry in the probabilities of future values of x_{t+j} as the discrepancy λ increases. (A) and (B) are very important restrictions on beliefs in RBE since in situations as postulated here we can say that all rational agents believe with certainty that an event will happen and *the only difference among them is the time at which it will occur*.

Note that property (B) states that the rise in the probability of reversion is typically only on average over time and does not necessarily occur at date t which is the date at which x_t is above or below its stationary forecast. Hence, stationarity reversion is a more general phenomenon than the now familiar "mean reversion". This implies that the RBE theory of asset prices is compatible with prices exhibiting

mean reversion. Moreover, mean reversion is a strong sufficient condition for serial correlation of returns but serial correlation of asset returns can be present in other price processes which satisfy only stationarity reversion, not mean reversion. The important conclusion which we stress is that *serial correlation of equilibrium asset returns is compatible with the RBE theory of asset pricing.* Moreover, a typical RBE will exhibit some form of reversion to a central tendency and serial correlation of returns will result. Hence, the mounting empirical evidence in support of the presence of serial correlation in asset returns (see for example Fama and French [1988], Poterba and Summers [1988] and Campbell and Shiller [1988b]) is indirect empirical evidence in support of the RBE asset pricing theory. We incorporate these ideas in the empirical evaluation below.

The theory of rational beliefs implies certain orthogonality and correlation conditions which are related to "stationarity reversion". Thus consider the process of observations $\{x_t, t = 1, 2, 3, \ldots\}$, $x_t \in \mathbb{R}^N$ with true probability Π and for a measurable function $g : \mathbb{R}^N \to \mathbb{R}$ consider the expression $g(x_{t+1}) = E_\Pi(g(x_{t+1})|I_t) + \chi_{t+1}$. Since Π is the probability of the process we conclude that the expectations of the χ_t and their time average are zero. In addition, we have the basic Π orthogonality of χ_{t+1} to the subspace of random variables generated by I_t. In an RBE agents form beliefs about the observables and thus if Q is such a belief and we write

$$g(x_{t+1}) = E_Q(g(x_{t+1})|I_t) + v_{t+1} \tag{15}$$

then we have the standard result that $E_Q(v_{t+1}|I_t) = 0$ and

$$\mathrm{Cov}_Q(v_{t+1}, x_{t-j}) = 0 \text{ for any } j \geqq 0. \tag{16}$$

(16) implies that

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} [x_{t-j} - E_Q(x_{t-j}|I_{t-j})][v_{t+1}] = 0 \quad Q \text{ a.e. } j \geqq 0. \tag{17}$$

It is important to note that (17) holds Q a.e. but not necessarily Π a.e. It can, however be shown that if the dynamical system under consideration is ergodic and if the belief Q satisfies the two rationality axioms of Kurz [1994a] then (17) holds Π a.e. as well. Although v_{t+1} may depend upon variables which are in the information set I_t, it is still true that *the time average of these functions does average out to 0 Π a.e.* These conclusions crucially depend upon the assumption that g is a time invariant function.

The situation becomes much more complicated when in (15) we have a sequence of time dependent functions g_t which would arise in a typical optimization such as (8a)–(8b) when the belief of the agents incorporate any non-stationarity. Conditions like (8b) imply that there exist functions $\bar{\eta}_{t+1}^k \neq 0$ which are not orthogonal to the subspace spanned by I_t such that

$$\frac{u_k'(c_{t+1}^k)}{u_k'(c_t^k)} \left(\frac{P_{t+1} + r_{t+1}}{P_t} \right) \frac{1}{1+\delta} - 1 = \bar{\eta}_{t+1}^k. \tag{18}$$

The decisions of agent k may be time dependent. Hence, one representation of the error in (18) is

$$\bar{\eta}_{t+1}^k = f_t^k(I_t) + \varepsilon_{t+1}^k \tag{19}$$

where the function f_t^k represent the mistakes of agent k. Here, however, the time average of the right hand side of (19) may not converge Π a.e. and even if it converges, it may not converge to zero. A special case of the functions f_t^k arises from the serial correlation of the forecast errors which make the error depend upon the entire past history. The presence of functions like f_t^k which violate the standard orthogonality conditions of the Market Efficiency Theory is the primary tool by which we can empirically demonstrate the presence of endogenous uncertainty and thus support the validity of the RBE theory of asset pricing. However, our discussion here shows that if such a sequence of functions is empirically established then the crucial restriction which the theory imposes on the sequence is that generally we should expect that $f_t^k(\cdot) \neq f_\tau^k(\cdot)$ for $t \neq \tau$. We return to examine these issues in the econometric specifications below.

2.2.E The number of possible RBE

We postulated that the dividend process $\{r_t, t = 1, 2, \ldots\}$ is fixed and now suppose that the utility functions are C^2. Hence, for each set of Rational Beliefs $(Q^1, \ldots, Q^K)$ our financial economy is a regular economy and it has a finite or a countable number of market clearing solutions. In all that we do we postulate that there is a selection mechanism among those solutions so that in an RBE a particular market clearing solution is selected; we then incorporate this selection into the definition of the dynamical system. The question how many different RBE could be realized as equilibria with *different distributions* of beliefs is a different question. We assert that our model has a continuum of RBE. Since each RBE with a probability Π induces a stationary measure m, there are multiple equilibrium measures m and consequently multiple long-term averages of the endogenous variables. To see that this is so consider the effect of different distributions of beliefs $(Q^1, Q^2, \ldots, Q^K)$. If the utility functions of each of the agents is "future oriented" i.e. that it has the property that his choice functions are sensitive to his probability beliefs about future opportunities, then different configurations of beliefs will induce different demand functions $((\theta_t^1, B_t^1), (\theta_t^2, B_t^2), \ldots, (\theta_t^K, B_t^K))$ for assets and hence different equilibrium prices. We can imagine two economies which are identical in all respect except for the distribution of beliefs of the agents. This difference may result in the two economies having different asymptotic distributions of the endogenous variables. In economy 1 asset prices may have low volatility relative to dividends while in economy 2 their volatility may be much higher. The time average of asset prices $\bar{P}$ may also not be the same in the two economies. These are exactly the variations among the continuum of RBE exhibited in Kurz [1994b] where we study a simple economy with linear demands and quadratic cost.

2.2.F Example: Shifting regimes and asymptotic correlation

The example aims to highlight some of the essential features which we employ in the empirical evaluation of Section (3). We select a sequence of dates $\{\tau_1 = 1, \tau_2, \tau_3, \ldots\}$ which are *the dates of regime changes*. We let $l_k = \tau_{k+1} - \tau_k$ for $k = 1, 2, \ldots$ and require l_k to be distributed on, say, the L integers $\{1, 2, \ldots, L\}$. l_k is the *duration of*

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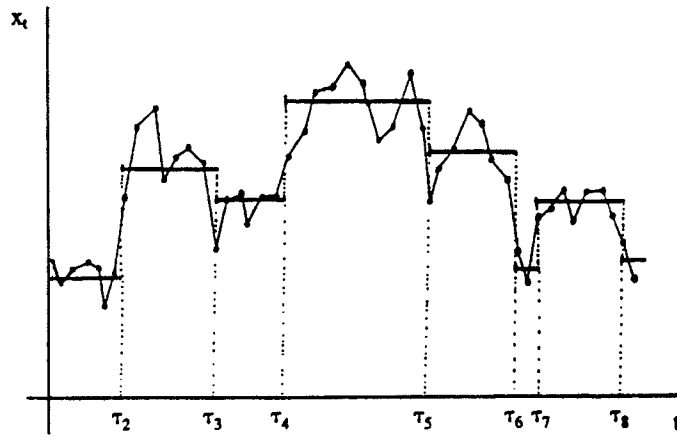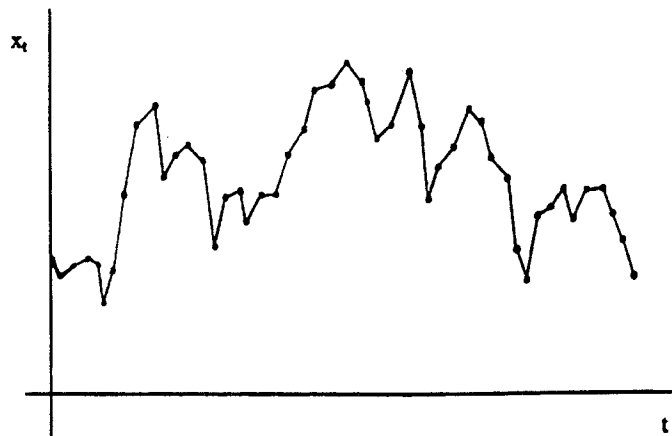in F
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Figure 2

Figure 3

regime k and in this example the duration is a random variable with maximum length of L periods. Now the sequence of real numbers $\{\Upsilon_j, j = 1, 2, \ldots\}$ is selected such that the stochastic process $\{x_t, t = 1, 2, \ldots\}$ is stable and satisfies $x_t = \Upsilon_{k(t)} + \varepsilon_t$ where $k(t) = k$, and k satisfies $\tau_k \leq t < \tau_{k+1}$. The sequence $\{\varepsilon_t, t = 1, 2, \ldots\}$ is an i.i.d. random noise with mean 0. A typical sample path looks as in Figure 2 where the levels of the horizontal bars are the values of $\Upsilon_{k(t)}$. Agents do not know the postulated structure and therefore they see the data as in Figure 3. The interest naturally focuses on the autocorrelation in the data. It is to be noted that although the x_t are independent, the stationary measure m is computed by averaging across the regimes and *this will, typically, induce autocorrelation of the x_t around the mean $\bar{x}$.*

This example brings up an important principle related to the use of information for trading securities in a world of shifting regimes which is the basis for our

empirical work in Section (3). The central problem of agents in such a market is how to weigh the value of the most recent data in contrast to the long term statistics represented by the stationary measure m. An agent whose belief is m, the stationary measure, has a simple answer: only long term statistics should be consulted in modeling the investment decision. An agent in our stock market economy who believes that the economy is characterized by a sequence of shifting regimes has a different perspective. If he believes that each regime lasts long enough to provide some data on the basis of which one can draw some inference, he rejects the validity of the long term correlations implied by m and puts heavier weight on the most recent data. In fact, such an agent believes that all opportunities for large excess returns are associated with the timing and nature of the turning points of the process. He then attempts to forecast the changes or, at least, identify then after they took place by seeking any data that would support the hypothesis that a new regime is in place. But then his "within regime" learning entails the statistical problem of estimating the starting date and nature of any regime based on a *small sample* of observations generated by the operating regime while utilizing the long term statistics only as a stability benchmark. The more data is required by the agent to make a decision the less risk is taken and correspondingly reduced are the profit opportunities. All important decisions in such a market must be made with small samples and the risk of investing in such an environment is the risk of drawing the wrong inference from *small* samples! We shall see that even if such agents learn $\Upsilon_{k(t)}$ after the regime is well established, ex-post correlation between their forecast innovations and other observables reveals their inability to accurately forecast the dates and magnitude of the changes.

3 The empirical evaluation

An extensive econometric literature has investigated during the last 25 years the movement of stock prices and returns. Early investigators followed the seminal work of Fama in the 1960s which is represented in Fama [1970]. This work aimed to show that capital markets are efficient. A major controversy related to the issue of excess volatility originated in the work of LeRoy and Porter [1981] and Shiller [1981]. The most recent work explores the important question of the predictability of returns and its relationship to the problem of excess volatility and the present value hypothesis (for example, see the interesting papers of Fama and French [1987], [1988], Poterba and Summers [1988] and Campbell and Shiller [1988a], [1988b]). Most of the work since the 1960s examines the regularity of the data on stock prices, dividends, interest rates, etc. over very long periods covering the 19th and the 20th centuries. In estimating stationary models which utilize long time series these studies, in effect, estimate the parameters of the *stationary* measure which we discussed in Section (2.1). As explained at the end of Section 2, we assume a non-stationary economy but in order to enable an empirical evaluation we postulate a drastically simplified form of non-stationarity represented by a stable sequence of regimes or, what we shall call, *environments*. Each environment is represented by a fixed economic and technological structure covering an episode of economic growth over a number of years. Although unpredictable in the statistical sense, such

behavior can be well explained ex-post by the economic factors operating within each environment and which may not be operative in earlier or later subperiods. Since environments are different from each other discontinuous changes take place at the dates of change which we refer to as the "break points". We study the post war period and specify *exogenously* the three environments with which we work. The first is the *expansionary* environment of 1947:1–1965:4; the second is the environment of *retrenchment* 1966:1–1981:4 and the third is the *restructuring* environment of 1982:1–1992:3. In the next section we shall provide the justification for these dates.

The representation of non-stationarity as a sequence of "regimes" or "environments" is a modeling approximation: it is certainly not new and has a long history in economics. A partial list of *recent* prior works which either employ or discuss this device would include Benerjee, Lumsdaine and Stock [1990], Christiano [1988], Cooper and Durlauf [1993], DeLong and Summers [1986], Diebold and Rudebusch [1992], Hamilton [1989], Mankiw, Miron and Weil [1987], Neftci [1984], Perron [1989], [1990], Potter [1991], Romer [1990], [1991] and Zarnowitz and Moore [1986].

This work is also related to several earlier papers which we mention in order to clarify our approach. Perron [1989] seeks exogenous events to locate breaks in the U.S. aggregate output time series. Treating the oil shock of 1973 as exogenous, he locates the break in 1973. Perron thus rejects the hypothesis of stationary increments in favor of a vision of non-stationarity represented by discrete breaks in the structure of the process. Our economic conclusions regarding the post war era agree with Perron's although we do not assume an exogenous cause for the breaks. We place the changes in 1966 and 1982 and do not think that exogenous events caused them. A second related paper is Mankiw, Miron and Weil [1987] which studies the impact of the founding of the Federal Reserve system in 1914 on the behavior of equilibrium interest rates. We differ with Mankiw, Miron and Weil [1987] in that we study a *sequence of structural changes* and do not presume them to be caused by exogenous events. We also note that Christiano [1988] and Benerjee, Lumsdaine and Stock [1990] show that results of testing hypotheses regarding breaks in the data can be misleading if the break points are dependent upon the data *which is being analyzed*. We provide extensive historical reasoning for our exogenous selection of the break points. In Section (3.2) we also provide independent empirical support for the selected break points and this empirical work *uses no data on stock returns* which is the data for whose analysis we specified the break points to begin with.

3.1 The historical perspective

At the end of World War II the U.S. economy reached a level of world predominance unparalleled in modern times. For an extended period after the war it experienced a very high rate of development: U.S. technology was internationally dominant; the motivation and vitality of U.S. entrepreneurship and labor force were at a very high level. American products were dominant in world markets facing only minor foreign competition, and the U.S. was heavily engaged in profitable investments abroad. It is fair to say that for more than a generation after the war the U.S. experienced

extraordinary prosperity with little challenge to its economic supremacy. All these facts are so well documented that it is hardly necessary to further elaborate on them. What we now want to stress is the converse: a major change occurred in the mid 1960's and these favorable conditions began to unravel. Without a detailed historical analysis let us briefly state that in our view there are two central factors which account for the change. First, the success of the U.S. economy generated a rising wave of political claims on the social output. This resulted in a growing pressure on wages as well as political demands for the "elimination" of poverty. Despite the wide ranging incentive effects involved, the process culminated in the 1965 legislation of the Great Society. It dramatically expanded the legal basis of a system of economic entitlements that did not call for corresponding tax increases. The same misguided sense of economic abundance led President Johnson to commit the U.S. in July of 1965 to a full scale land war in Vietnam without raising taxes.

The private sector provided the second and symmetric side of the great changes in the mid 1960s. The economies of Europe and Japan completed by the mid 1960s their post-war reconstruction and began to challenge the U.S. manufacturing sector on a very broad front. Industries such as steel products, shipbuilding, machine tools, small automobiles and electric appliances began to face significant international competition leading the *growth rate* of U.S. manufacturing output to peak in 1965 and to decline thereafter. Manufacturing output grew by 53.99% between 1955 and 1965 but by only 26.76% between 1965 and 1975. We stress that 1965:4 was not a time of recession; such a downturn did not occur until 1970. This competitive challenge to U.S. manufacturing in the mid 1960s was launched with fully modernized plant and equipment and with newly developed methods of organizing production. In comparison, a significant portion of the plant and equipment employed by the U.S. manufacturing sector was aging and it's organizational methods becoming obsolete. Equally important, a significant portion of the human capital of the manufacturing labor force became obsolete almost overnight.

It is a significant fact that the private sector did not respond rapidly and decisively to the new environment. In some industries such as appliances and certain steel products U.S. manufacturers simply withdrew from the market. In others, such as automobiles, the domestic producers yielded part of the market (e.g. small cars) and made no adjustments in the rest of the product line. These facts are compatible with the sense of resource abundance and invincibility demonstrated by the public sector and by the society at large. The net merchandize trade balance of the U.S. reached a high point of $6.801 billion in 1964 and declined after that. It turned negative in 1971 and went on to become dramatically negative in later years. Our hypothesis is that the best date to approximate the changed conditions is the quarter of 1966:1. We place it after the completion of an extraordinary avalanche of the Great Society legislation in 1965 and after the U.S. is fully committed to a land war in Vietnam. Thus we choose these historical and political landmarks to exogenously identify or signal the change; they are not the *cause* of the change. This procedure will remain our guiding principle in the selection of the second breaking point.

It took many years before the significance of the changes discussed above was recognized. It also took some time before the retrenchment in manufacturing

expanded to other sectors of the U.S. economy. Moreover, other factors such as the two oil shocks of the 1970s contributed to what has since become known as the "productivity slowdown" in the U.S. A great deal has been written about the slowdown and we cannot review here its diverse aspects[3]. From our perspective the political and economic environment underwent an important transformation in 1981 and early 1982 whose main thrust was to reverse the forces put in motion in the mid 1960s. First, a specific public policy and a radically different tax structure were put in place which aimed to restrict economic entitlements and seek to revitalize incentives. Second, both public policy as well as the private sector accelerated the efforts to restructure the weaker components of the manufacturing sector and to rapidly expand the high-technology industries where a technological revolution was taking place. The objective became to raise productivity and face the challenge of a global economy. We do not evaluate the success of this effort[4]. Here we simply postulate that a new environment was being established early in the 1980s. Our hypothesis places the break point in 1982:1. Again we place it at the *end of the first year of the new administration after the massive legislative program of 1981 was completed.*

Our empirical work below will consist of two parts. The first part is a brief analysis of structural change in U.S. manufacturing which aims to show that our assumption of two breaks in 1966:1 and 1982:1 is supported by *independent data which is unrelated to the data on stock prices or returns.* This bolsters the geopolitical considerations outlined above; it is presented in response to the views of some writers expressed earlier that only the data under analysis should determine the break points. Since our theory predicts environments which may not last long enough to provide sufficient data, considerations of efficiency compel us to seek independent methods to support the selected dates for the breaks. The second and central part of this paper is the estimation of a structural model of asset prices in support of the RBE asset pricing theory.

3.2 Structural change in U.S. manufacturing: A verification of the two breaks hypothesis

Without using any data on stock returns (which is the object of our analysis in the next section) we now test the hypothesis that structural breaks occurred in the U.S. manufacturing and foreign trade sectors in 1966:1 and 1982:1. To do this we examine the 1947–1991[5] quarterly time series of four variables; merchandise exports (Ex),

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[3] See, for example, Northsworthy, Harper and Kunze [1979]. Papers by P.K. Clark and R.J. Gordon in the same issue (1979:2) of the *Brookings Papers on Economic Activity* also report on different aspects of the same problem.

[4] It is very likely that as of now, at the end of 1995, this environment is still in place.

[5] All the empirical work in this paper cover the period 1947:1–1992:3 except for this section. The work here was based on an index of manufacturing output received from the Federal Reserve (thanks to Suzanne Cooper) which was not seasonally adjusted. This covered the period 1947:1–1991:2. We later switched to the seasonally adjusted index reported in our basic file obtained from Citibank and this data covers the period 1947:1–1992:3. Since our results are entirely insensitive to this correction we decided to save the extra work and report the results in this Section (3.2) for 1947:1–1991:2.

merchandise imports (Im), profits of the manufacturing sector (Mp) and an index of manufacturing output (Mf). Nominal values of exports, imports and manufacturing profits were deflated by the consumer price index. We then estimate a system of four equations of the following AR form

$$\log x_t = \sum_{j=0}^{2} (\beta_0^j + \beta_1^j t)E_t^j + (\beta_2 Q_2 + \beta_3 Q_3 + \beta_4 Q_4) + z_t$$

$$z_t = \sum_{j=0}^{2} \sum_{n=1}^{N} \rho_n^j z_{t-n} E_t^j + \varepsilon_t$$

where

x_t – any one of the four variables specified,

$$E_t^0 = \begin{cases} 1 & \text{if } 1947{:}1 \leq t \leq 1965{:}4 \\ 0 & \text{otherwise,} \end{cases}$$

$$E_t^1 = \begin{cases} 1 & \text{if } 1966{:}1 \leq t \leq 1981{:}4 \\ 0 & \text{otherwise,} \end{cases}$$

$$E_t^2 = \begin{cases} 1 & \text{if } 1982{:}1 \leq t \leq 1991{:}2 \\ 0 & \text{otherwise,} \end{cases}$$

(Q_2, Q_3, Q_4) are quarterly dummies,
(β_0^j, β_1^j) are the trend parameters of environment $j, j = 0, 1, 2$,
$(\rho_1^j, \rho_2^j, \dots, \rho_N^j)$ are the AR parameters of environment $j, j = 0, 1, 2$,
ε_t – i.i.d noise common to all environments.

For the Ex, Im, and Mp equations $N = 2$ was sufficient to eliminate the autocorrelation in the residuals. For Mf, AR(6) was needed. Estimation was done using non-linear least squares techniques. With 2 break points we estimate 48 parameters. Given the large number of parameters we omit this information[6]. Our main interest here is in tests for structural change. All Wald tests presented below relate to the equality of $(\beta_0^j, \beta_1^j, \rho_1^j, \dots, \rho_N^j)$ across $j = 0, 1, 2$. The "two-breaks" hypothesis is that the three vectors $(\beta_0^j, \beta_1^j, \rho_1^j, \dots, \rho_N^j)$ are different. A "one break" hypothesis specifies that an appropriately chosen vector is different than the others.

Table 3 provides a summary of a set of tests of the two breaks hypothesis. It gives the p values for accepting the two breaks hypothesis. We test all interesting components of the four equations: subsystems of equations, AR parameters only, trend parameters only and finally all parameters combined. Consider, for example, the last row in the table which reports the results for the single equation Mf. *Assuming that the rest of the equations are estimated with two breaks* we then test the contribution of varying the specifications of the Mf equation as indicated. We accept the two breaks hypothesis with significance level of .0001 with respect to the AR parameters, of .0300 with respect to the trend parameters and of less than .0001 with

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[6] The estimated parameters, the t-statistics and R^2 are provided in the appendix of Kurz [1994c] which is an earlier version of this paper.

Table 3. p values for the test of two breaks in 1966:1 and 1982:1

Equation subset tested	AR parameters only	Trend parameters only	All parameters
Ex, Im, Mp, Mf	.0000 ($\chi^2_{24} = 66.7$)	.0000 ($\chi^2_{16} = 112.6$)	.0000 ($\chi^2_{40} = 185.0$)
Ex, Im, Mp	.0057	.0000 ($\chi^2_{12} = 99.7$)	.0000 ($\chi^2_{24} = 138.8$)
Ex, Im, Mf	.0000 ($\chi^2_{20} = 64.0$)	.0000 ($\chi^2_{12} = 106.0$)	.0000 ($\chi^2_{32} = 179.6$)
Mp, Mf	.0003	.0141	.0000 ($\chi^2_{24} = 56.4$)
Ex	.0024	.0518	.0346
Im	.0081	.0000 ($\chi^2_{4} = 68.5$)	.0000 ($\chi^2_{8} = 91.8$)
Mp	.4472	.0022	.0292
Mf	.0001	.0300	.0000 ($\chi^2_{16} = 48.7$)

Note: Whenever the p value is negligible, Table 3 provides the approximate χ^2 value and the associated degrees of freedom.

respect to all the parameters of the manufacturing output equation combined. A similar interpretation is given to all the rows. It is clear from the table that for any subset of equations there is some subset of parameters for which the two breaks hypothesis is accepted with confidence higher than 99%.

We turn next to examine the question of the "one break" against the "two breaks" hypotheses. Table 4 provides a summary of our results. It shows that the "two breaks" hypothesis is strongly accepted against either the "no breaks" or the "one break" hypotheses.

The tests presented would not be valid if the variance of any one of the variables under study in any of the environments is infinite. The condition that ensures this is that the roots of the characteristic polynomials (for each equation when $\hat{\rho}^j_n$ is the estimate of ρ^j_n in the basic model) $1 - \sum_{n=1}^{N} \hat{\rho}^j_n y^n = 0$ for $j = 0, 1, 2$ are all outside the unit circle. Our calculations show that, indeed, all the 36 roots are larger than 1 in absolute value.

Table 4. p values for the test of one or two breaks

Hypothesis to be tested:

Date of breaks	Against	Entire system	AR parameters only	Trend parameters only
1966:1	No Breaks	.0000 ($\chi^2_{20} = 71.0$)	.0000 ($\chi^2_{12} = 54.7$)	.0697
1982:1	No Breaks	.0000 ($\chi^2_{20} = 56.7$)	.0000 ($\chi^2_{12} = 43.0$)	.0150
1966:1 and 1982:1	No Breaks	.0000 ($\chi^2_{40} = 185.0$)	.0000 ($\chi^2_{24} = 66.7$)	.0000 ($\chi^2_{16} = 112.6$)
1966:1 and 1982:1	1982:1 Only	.0000 ($\chi^2_{20} = 122.6$)	.0699	.0000 ($\chi^2_{8} = 82.7$)
1966:1 and 1982:1	1966:1 Only	.0000 ($\chi^2_{20} = 77.4$)	.0008	.0000 ($\chi^2_{8} = 66.3$)

Note: Whenever the p value is negligible, Table 4 provides the approximate χ^2 value and the associated degrees of freedom.

To conclude this section let us note first that results like those presented here provide support for a process of belief formation which allows for structural change. Many economic agents would conclude that, indeed, *it is possible that the manufacturing and foreign trade sectors of the U.S. experienced structural changes in the post-war period.* Equally important, the results *provide an independent support for our specified dates of breaks.*

3.3 Structural change, endogenous uncertainty and asset returns

3.3.A Theoretical considerations

We turn now to the asset pricing model presented in Section 2 above. To do this we modify the model in two ways. First, we shall employ only condition (8b) and ignore the optimal borrowing equation (8a). This enables us to sidestep the difficult empirical problem of identifying a risk free asset. Second, we need to explain in greater detail how to incorporate into the first order conditions (8b) the process of structural change represented by the sequence of environments. To do this we postulate that there exists an infinite number of different possible environments which we describe by $\mathcal{L} = \{E^1, E^2, E^3, \ldots\}$. These should be thought of as a collection of parameter vectors or structural relations. A random process $\{E_t, t = 1, 2, \ldots\}$ on $\mathcal{L}$ generates a sequence of realizations where we denote by E_t^j the identity of the environment realized at date t. These random variables are not observable by anyone. As in Section 2 the observable variables are $x_t \in \mathbb{R}^N$ and therefore we need to think of the true stochastic process of the economy as represented by a sequence of pairs $\{(x_t, E_t), t = 1, 2, \ldots\}$ and the true equilibrium probability Π defined on subsets of $(\mathbb{R}^N \times \mathcal{L})^\infty$. This formulation specifically assumes that the random process $\{E_t, t = 1, 2, \ldots\}$ on $\mathcal{L}$ is endogenous since E_t and x_t are stochastically interdependent and many of the variables in x_t are endogenous variables. We shall postulate, however, that *each environment remains in effect a reasonably long period* and the dates of switch are random. To highlight the non-stationarity of the process $\{(x_t, E_t), t = 1, 2, \ldots\}$ we postulate that *an environment which is realized is never visited again*[7] and since every environment is new and different, it is not statistically predictable.

Although the random variables $E_t \in \mathcal{L}$ are not observable, the agent's beliefs Q^k for all k incorporate joint probabilities on both observable variables as well as unobservable environments. We denote by $Q_t^k(A | E_t^j)$ the conditional probability of A given E_t^j as perceived by k at date t and by $Q_t^k(E_{t+1}^j | I_t)$ the probability of E_{t+1}^j given I_t as perceived by k at date t. With agent's beliefs constituting joint probabilities between data and unobserved environments, the conditional probabilities $Q_t^k(E_{t+1}^j | I_t)$ incorporate "within regime" learning which was discussed in Section (2.2.F). Given this specification of our model we can then rewrite condition (8b) in the form

$$\sum_{j=1}^{\infty} E_{Q_t^k} \left[\frac{u_k'(c_{t+1}^k)}{u_k'(c_t^k)} \frac{P_{t+1} + r_{t+1}}{P_t} \Big| I_t, E_{t+1}^j \right] Q_t^k(E_{t+1}^j | I_t) = 1 + \delta. \tag{20}$$

---

[7] Contrasted with modeling the environment as a stationary process. See for example Hamilton [1989].

This implies that there exists a stochastic process $\{\eta_t^k, t = 1, 2, \ldots\}$ such that for each k and each t the following conditions hold:

$$\frac{u_k'(c_{t+1}^k)}{u_k'(c_t^k)} \left(\frac{P_{t+1} + r_{t+1}}{P_t} \right) \frac{1}{1+\delta} = \eta_{t+1}^k \qquad (21\text{a})$$

where c_{t+1}^k, P_{t+1}, r_{t+1} and η_{t+1}^k depend upon the realized E_{t+1} and c_t^k and P_t upon E_t,

$$E_{Q_t^k}(\eta_{t+1}^k) = 1, \qquad (21\text{b})$$

$$\text{Cov}_{Q_t^k}(\eta_{t+1}^k, x_\tau) = 0 \text{ for any observable } x_\tau \text{ with } \tau \leq t \text{ in the information } I_t. \qquad (21\text{c})$$

The above conditions are standard except for the crucial fact that they are defined with respect to Q^k and not Π. Thus, as explained in detail in (2.2.B)–(2.2.D), for most agents we must have

$$E_{\Pi_t}(\eta_{t+1}^k) \neq 1, \qquad (22\text{a})$$

there exist variables y_τ $\tau \leq t$ in I_t such that $\text{Cov}_\Pi(\eta_{t+1}^k, y_\tau) \neq 0$. $\qquad (22\text{b})$

This brings us to the specification of the econometric implications of the theory of rational beliefs.

3.3.B Econometric implications of the theory of rational beliefs

To review the orthogonality and empirical implications of the theory consider again conditions (6a) and (21a). Condition (6a) specifies the equilibrium price function $P_t = \varphi_t(\cdot)$ as a function of the history and the perceived environment at t. Thus, one must think of P_t as a function of all past and present variables at t. In (21a) we specify η_{t+1}^k as the forecasting innovation[8] of agent k and although briefly explained in (18)–(19), we would risk some repetition and explain their structure in detail. Thus suppose that P_{t+1} depends upon some variable x_τ, $\tau \leq t$ which is in I_t. If agent k knows this fact and *if he also knows how* x_τ *enters the function* $\varphi_{t+1}(\cdot)$ he will correctly condition his forecast on this variable and adjust his optimal behavior accordingly. This will assure that all the information about x_τ will be incorporated into the decision functions of the agent and none will "enter" into the innovation η_{t+1}^k. If, however, agent k does not know that x_τ is an important variable or if he does not know exactly how x_τ enters the function $\varphi_{t+1}(\cdot)$, then he will condition on the wrong function and make the mistake of thinking that η_{t+1}^k does not depend upon x_τ. This is the mechanism by which x_τ "enters" the innovation η_{t+1}^k. The same argument applies to all variables relative to which the conditional probabilities of Π_t and Q_t^k differ, representing mistakes of the agents. It is thus seen that the mistake of an agent is of crucial empirical implication: it identifies that component of η_{t+1}^k which is explainable by those past variables on which the agent failed to correctly condition his probability assessment. What are then the restrictions imposed by the theory on η_{t+1}^k?

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[8] Recall our distinction in (7) between a forecast "error" and a forecasting "mistake". The innovation captures both of these.

To answer this last question let us decompose η^k_{t+1} into the three components

$$\eta^k_{t+1}(x^t) = M^k_{t+1}(x^t)\hat{\zeta}^k_{t+1}\hat{\varepsilon}_{t+1} \qquad (21d)$$

where $\hat{\varepsilon}_{t+1}$ is the positive exogenous random shock; $\hat{\zeta}^k_{t+1}$ is the positive measure of pure (i.e. that it is not a function of observables at t) local bubble and amplification of k while $M^k_{t+1}(x^t)$ is the *systematic* mistake of k which is defined over the history x^t. Both $\hat{\zeta}^k_{t+1}$ and $\hat{\varepsilon}_{t+1}$ are, by definition, orthogonal to the subspace of random variables spanned by the information at t and are uncorrelated with each other. The crucial requirement explained in (2.2.D) is that the functions $M^k_{t+1}(\cdot)$ *should not be time invariant and their differences unpredictable*. Thus we have the following conclusions:

(i) the functions $M^k_t(\cdot)$ are not time invariant and their differences unpredictable;

(ii) $\displaystyle\lim_{T\to\infty} \frac{1}{T}\sum_{t=1}^{T} \hat{\zeta}^k_{t+1} = \lim_{T\to\infty}\sum_{t=1}^{T}\hat{\varepsilon}_{t+1} = 1$

These two conditions will be further clarified when we specify the econometric parametrization. We may, however, note now that from the point of view of testing for the presence of endogenous uncertainty the terms $\hat{\zeta}^k_{t+1}\hat{\varepsilon}_{t+1}$ clearly contain a component of pure, endogenous, local bubbles and amplification. In this paper we do not attempt to isolate this component. On the other hand, *the random functions* $M^k_{t+1}(x^t)$ *constitute pure and measurable endogenous uncertainty* since they arise strictly because of the nature of the mistakes of the agents. It then follows that accepting the empirical hypothesis that there exists a component of the utility returns which is explainable by past variables *is equivalent to demonstrating that endogenous uncertainty is present in the market.*

In the model below we parametrize the non-stationarity of the economy by a sequence $\{E_t,\ t = 1, 2, \ldots\}$ of shifting environments and test the hypothesis that such structural changes explain some of the observed variability of returns. Recall that had agents been able to perfectly predict the nature and the changes of the environments, these structural changes could not explain any of the variability of returns. It is then a correct reasoning to conclude that *it is the failure of the agents to perfectly predict these structural changes that results in the fact that structural changes explain some of the variability of returns!* Moreover, due to stationarity reversion we would expect to find serial correlation in the returns and this we shall directly test.

The estimation of the parameters of (21a) requires individual data which we do not have and this necessitates the utilization of aggregate data. In order to enable aggregate parametrization we introduce the following simplifying assumption:

Assumption: For all agents

$$u_k(c) = \frac{A}{1-\gamma}c^{1-\gamma}. \qquad (23)$$

From (20) and (21) we have that

$$(c_{t+1}^k)^{-\gamma} \left(\frac{P_{t+1} + r_{t+1}}{P_t} \right) = (1 + \delta)(c_t^k)^{-\gamma} \eta_{t+1}^k$$

hence

$$c_{t+1}^k \left[\left(\frac{P_{t+1} + r_{t+1}}{P_t} \right) \frac{1}{1+\delta} \right]^{-1/\gamma} = c_t^k (\eta_{t+1}^k)^{-1/\gamma}.$$

We now aggregate over k and note that it is not likely that for a fixed t, η_{t+1}^k is correlated *across agents* with c_t^k since η_{t+1}^k reflects the beliefs of agents and there is no reason for these to be correlated with their past wealth. Moreover, even if such an aggregation effect was present *there is no reason to presume that it would change over time in any systematic way*. We thus assume it away and write

$$\sum_{k=1}^K c_{t+1}^k \left[\left(\frac{P_{t+1} + r_{t+1}}{P_t} \right) \frac{1}{1+\delta} \right]^{-1/\gamma} = \left(\sum_{k=1}^K c_t^k \right) \frac{1}{K} \sum_{k=1}^K (\eta_{t+1}^k)^{-1/\gamma}.$$

Hence

$$\frac{P_{t+1} + r_{t+1}}{P_t} = (1 + \delta) \left(\frac{c_t}{c_{t+1}} \right)^{-\gamma} \eta_{t+1} \tag{24a}$$

where

$$\eta_{t+1} = \left[\frac{1}{K} \sum_{k=1}^K (\eta_{t+1}^k)^{-1/\gamma} \right]^{-\gamma} \quad \text{and} \quad c_t = \sum_{k=1}^K c_t^k. \tag{24b}$$

From (21d) we have that

$$\eta_{t+1} = \left[\frac{1}{K} \sum_{k=1}^K (M_{t+1}^k(x^t))^{-1/\gamma} (\hat{\zeta}_{t+1}^k)^{-1/\gamma} \hat{\varepsilon}_{t+1}^{-1/\gamma} \right]^{-\gamma}$$

$$= \left[\left(\frac{1}{K} \sum_{k=1}^K (M_{t+1}^k(x^t))^{-1/\gamma} \right) \left(\frac{1}{K} \sum_{k=1}^K (\hat{\zeta}_{t+1}^k)^{-1/\gamma} \right) \hat{\varepsilon}_{t+1}^{-1/\gamma} \right]^{-\gamma}$$

$$= M_{t+1}(x^t) \hat{\zeta}_{t+1} \hat{\varepsilon}_{t+1}$$

where $M_{t+1}(x^t) = \left(\frac{1}{K} \sum_{k=1}^K (M_{t+1}^k(x^t))^{-1/\gamma} \right)^{-\gamma}$ and $\hat{\zeta}_{t+1} = \left(\frac{1}{K} \sum_{k=1}^K (\hat{\zeta}_{t+1}^k)^{-1/\gamma} \right)^{-\gamma}$. We now define $\varepsilon_{t+1} = \log \hat{\zeta}_{t+1} + \log \hat{\varepsilon}_{t+1}$ and consequently $\log \eta_{t+1} = \log M_{t+1}(x^t) + \varepsilon_{t+1}$. Finally, define

$$R_{t+1} = \log \frac{P_{t+1} + r_{t+1}}{P_t}$$

$$z_{t+1} = \log \eta_{t+1} \tag{24c}$$

and we can conclude that

$$R_{t+1} = \log(1 + \delta) + \gamma \log \left(\frac{c_{t+1}}{c_t} \right) + z_{t+1}. \tag{25}$$

To complete the specification we need to apply our earlier analysis to the equilibrium quantity z_{t+1} which is the logarithm of the sum of individual terms $(\eta_{t+1}^k)^{-1/\gamma}$. We shall first specify the parametrization of z_{t+1} and then explain what are the restrictions (and hypotheses to be tested) which are imposed upon it by the theory of rational beliefs. Since we postulate three different environments we shall specify z_{t+1} to be the sum of three prediction functions and a pure error term. That is, using the notation of the three dummy variables (E_t^0, E_t^1, E_t^2), we postulate that

$$z_{t+1} = f_t^0 E_t^0 + f_t^1 E_t^1 + f_t^2 E_t^2 + \varepsilon_{t+1} \tag{26a}$$

where for $j = 0, 1, 2$

$$f_t^j = \alpha_1^j + \alpha_2^j t + \alpha_3^j Y_{1t} + \alpha_4^j Y_{1,t-1} + \alpha_5^j Y_{2t} + \alpha_6^j Y_{2,t-1} + \alpha_7^j R_{t-1}. \tag{26b}$$

In accordance with the historical perspective outlined in Section (III.1) above, our hypothesis is that the factors which dominated the profitability of U.S. manufacturing in the post-war environments were the changes in productivity, growth-rates and the variability of the cost of basic commodities. Variables which provide good signals to these factors are defined by

$$Y_{1t} = \log \frac{P_{bt}}{P_{b(t-1)}} \quad \text{where } P_{bt} \text{ is the ratio of the CRB index of raw commodities to the CPI,}$$

$$Y_{2t} = \log \frac{M_{ft}}{M_{f(t-1)}} \quad \text{where } M_{ft} \text{ as in Section 3.2 above is an index of manufacturing output.}$$

Our theory imposes specific restrictions on the time pattern of the individual innovations $\eta_{t+1}^k(\cdot)$ and these properties are then inherited by the sequence $\{z_t, t = 1, 2, \ldots\}$ in (26a)–(26b). The assumption that the economy consists of a sequence of relatively fixed environments implies that at the end of each environment, its structure will be approximately known since agents can do in real time what we do in retrospect. We can therefore sum up the hypotheses implied by the theory as follows:

(A) *Within each environment returns are predictable and therefore endogenous uncertainty is present:* $f_t^0 \neq 0$, $f_t^1 \neq 0$ and $f_t^2 \neq 0$. A Wald test for the presence of endogenous uncertainty will test $\alpha^0 = \alpha^1 = \alpha^2 = 0$ where $\alpha^j = (\alpha_1^j, \alpha_2^j, \ldots, \alpha_7^j)$, $j = 0, 1, 2$.

(B) *Functions which predict returns cannot be constant: they must change over time and from environment to environment:* $f_t^0 \neq f_t^1 \neq f_t^2$. *Moreover, the prediction functions should not have a consistent pattern across environments: learning the forecasting functions of one environment should not improve forecasting in the next environment.* Wald tests for the time variability of the predicting functions are, in fact, tests for the presence of structural change. As in previous sections, a "two-breaks" hypothesis is specified as $\alpha^0 \neq \alpha^1$ and $\alpha^1 \neq \alpha^2$ whereas a "one-break" hypothesis must specify the break. Here we may have either $\alpha^0 \neq \alpha^1$ or $\alpha^1 \neq \alpha^2$ *but not both.* The variability of f_t^j across environments can take several forms in the specifications (26a)–(26b):

(i) a variable may be present in one environment but absent in another or it may have a different parameter in the two environments;

(ii) asset returns may be predictable in the different environments by different *patterns* at which the information becomes available. Thus, for example, an increase of the CRB index by 10% between $t-2$ and t could be interpreted differently in the two environments depending upon the allocation of these 10% between $\log \dfrac{P_{bt}}{P_{b(t-1)}}$ and $\log \dfrac{P_{b(t-1)}}{P_{b(t-2)}}$. This is why we allowed in (26a)–(26b) both (Y_{1t}, Y_{2t}) as well as $(Y_{1,t-1}, Y_{2,t-1})$;

(iii) the rate at which agents learn and adapt to each environment may not be the same. We parametrize this with the drift t and the lagged endogenous variable R_{t-1} allowing for serial correlation as explained in (II.2.D).

(C) Conditional upon (A) and (B), the random variables $\{\varepsilon_t, t = 1, 2, \dots\}$ are independent, mean 0 pure noise and where ε_t is orthogonal to all past variables at date t. Hence all the usual orthogonality conditions apply to these variables.

3.3.C Estimation

To estimate the model in (25), (26a) and (26b) we take R_t to be the logarithm of the real quarterly returns on the Standard and Poor's composite index during the 183 quarters of 1947:1–1992:3. However, even quarterly returns contains such large local bubbles that the consideration of longer term returns is warranted. We thus estimated the model with 1 and 2 year returns generated as a sum of the logarithms of quarterly returns in 4 and 8 (respectively) consecutive quarters. This creates overlapping data which will be properly accounted for in the estimation procedure. We note that Fama and French [1988], Poterba and Summers [1988] and Campbell and Shiller [1988b] consider long returns of up to 9–10 years. Most of their significant results apply to 3–5 years returns[9]. Thinking of t in terms of quarters we then define the one year return R_t^4 to be the sum of the logarithms of returns over t, $t-1$, $t-2$ and $t-3$. Similarly, instead of R_{t-1} in (26b) we use R_{t-4}^4. In the same manner Y_{1t}^4 and Y_{2t}^4 are now defined as $\log \dfrac{P_{bt}}{P_{b(t-4)}}$ and $\log \dfrac{M_{ft}}{M_{f(t-4)}}$. A corresponding redefinition of the variables for the two year returns is clear.

The model in (25), (26a), (26b) raises several other questions. First is the simultaneous equation bias in estimating γ. Within the estimation procedure we specify a set of instruments to estimate $\log \dfrac{c_t}{c_{t-J}}$, $J = 4, 8$ for the cases of 1 year ($J = 4$) and 2 year ($J = 8$) returns. The specified instruments include 4 lags of the logarithm of the J period consumption growth rate, two lags of the logarithm of the short term real rate on treasury bills and two lags of the logarithm of the real, 10 year, treasury bond rate. Hall [1988] argues that in the case of overlapping data, estimation of γ is

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[9] Cecchetti, Lam and Mark [1990] argue that the autocorrelation of long returns may be a result of small sample bias. Although we think their argument does not apply we note that the estimated bias in their own model (which is based on Hamilton's [1989] Markov switching mechanism) is negligible for 1 or 2 year returns.

biased when the timing of the instruments overlaps the timing of the dependent variable or the lagged dependent variable. To examine this question we shall consider the same set of instruments moved back in time more than 8 quarters so that their timing does not overlap the timing of R_t^4 and R_{t-4}^4. This would satisfy Hall's [1988] requirement for estimating (25), (26a), (26b) with one year returns and we shall report these results. As for the two year returns Hall's [1988] requirement implies a selection of instruments which are more than 4 years(!) back in time and this renders them essentially useless. Consequently we shall not attempt such estimation.

The second issue is raised with respect to the lags in the index of industrial production and the real value of basic commodity prices. We have already interpreted $(Y_{1,t}^J, Y_{2,t}^J)$ as the logarithm of the growth rates over $J = 4$ or $J = 8$ quarters. The model also specifies $(Y_{1,t-1}, Y_{2,t-1})$ and we interpret $(Y_{1,t-J}^J, Y_{2,t-J}^J)$ to be $\log \dfrac{P_{b(t-J)}}{P_{b(t-2J)}}$ and $\log \dfrac{M_{f(t-J)}}{M_{f(t-2J)}}$.

Next, for hypotheses testing it is much more convenient to rewrite (25), (26a), (26b) in the form of sequential environment *changes* rather than absolute levels. To do this define

$$e_t^1 = \begin{cases} 1 & \text{if } t \geq 1966{:}1 \\ 0 & \text{otherwise,} \end{cases} \qquad e_t^2 = \begin{cases} 1 & \text{if } t \geq 1982{:}1 \\ 0 & \text{otherwise.} \end{cases}$$

This implies that in (25), (26a)–(26b) we have $E_t^0 = 1 - e_t^1$, $E_t^1 = e_t^1 - e_t^2$ and $E_t^2 = e_t^2$. Inserting into (26a) we find that

$$\begin{aligned} z_{t+1} &= f_t^0(1 - e_t^1) + f_t^1(e_t^1 - e_t^2) + f_t^2 e_t^2 + \varepsilon_{t+1} \\ &= f_t^0 + (f_t^1 - f_t^0)e_t^1 + (f_t^2 - f_t^1)e_t^2 + \varepsilon_{t+1}. \end{aligned}$$

Hence the model that we end up estimating is formulated as follows. For $J = 4, 8$

$$R_{t+1}^J = \log(1 + \delta) + \gamma \log\left(\frac{c_{t+1}}{c_{t-J+1}}\right) + z_{t+1}^J \tag{27}$$

$$\begin{aligned} z_{t+1}^J &= \alpha_1 + \alpha_2 t + \alpha_3 Y_{1t}^J + \alpha_4 Y_{1,t-J}^J + \alpha_5 Y_{2,t}^J + \alpha_6 Y_{2,t-J}^J \\ &\quad + \alpha_7 R_{t-J}^J + \sum_{j=1}^{2} (\beta_1^j + \beta_2^j t + \beta_3^j Y_{1t}^J + \beta_4^j Y_{1,t-J}^J + \beta_5^j Y_{2,t}^J \\ &\quad + \beta_6^j Y_{2,t-J}^J + \beta_7^j R_{t-J}^J)e_t^j + \varepsilon_{t+1}. \end{aligned} \tag{28}$$

In (28) we interpret α_i as $\alpha_i = \alpha_i^0$ in (26a) and the β_i^j in (28) as the differences $\beta_i^1 = \alpha_i^1 - \alpha_i^0$, $\beta_i^2 = \alpha_i^2 - \alpha_i^1$ of the α_i^j in (26b). The constant $\log(1 + \delta)$ is not identifiable since we estimate only $\alpha = \alpha_1 + \log(1 + \delta)$. Thus, to test for endogenous uncertainty we will test (in (28)) the hypothesis that $(\alpha_2, \alpha_3, \dots, \alpha_7) = 0$ and $\beta^1 = \beta^2 = 0$. To test for structural change we test only $\beta^1 = 0$ or $\beta^2 = 0$ or both.

Finally, our estimation employs Hansen's [1982] Generalized Method of Moments (GMM) procedure as amended by White [1980] and Newey and West [1987]. It accounts for heteroskedasticity and the known serial correlation of the error term due to the overlapping data. Hence all the statistics and p values reported below incorporate these corrections.

   Table 5 reports the results of the estimation of (27)–(28). The first two columns of the table report the results for one year returns. Column 1 reports the case when the instruments are allowed to overlap in timing with the dependent variable or its lag and Column 2 reports the results when such overlap is not permitted. Column 3 reports the results for the two year returns.

   The first observation to be made about Table 5 is the large number of parameters which are statistically significantly different from 0; the Wald tests for the presence of endogenous uncertainty supports strongly this hypothesis. Moreover, the fraction of the variability of two year returns which is explained by variations in consumption growth is about 25%, leaving 75 percentage points to be accounted for by either endogenous or exogenous uncertainty. From Table 5 we can calculate that the variables specified in the model, excluding the constant and consumption, explain additional 48% of the variability of two-year returns. This means that in a division between these types of uncertainty, endogenous uncertainty accounts for at least 48/75 of the 75 percentage points which represent pure risk of returns. Since pure local bubbles may contribute some variability to the process $\{\varepsilon_t, t = 1, 2, \ldots\}$ we conclude that *endogenous uncertainty is the overwhelming component of risk in asset markets, accounting for more than 2/3 of all pure risks of asset returns.* We regard this conclusion to be of great significance both for economic theory as well as economic policy. From the point of view of economic theory, if some part of the risk of economic variables is propagated internally within the economy then the standard treatment of uncertainty in the Arrow-Debreu model is fundamentally flawed since it is based on uncertainty which is strictly exogenous. For such a reformulation of general equilibrium theory see Kurz [1974], [1993], Kurz and Schneider [1996], Kurz and Wu [1996], Svensson [1981], Henrotte [1996] and Nielsen [1996]. From the point of view of policy if economic fluctuations such as asset returns or business cycles are primarily endogenous phenomena, then public policy has an important impact on them. This is a subject for future research.

   Turning now to the crucial issue of structural change, our results are summarized in Table 6. The hypothesis that structural change is present in the returns is overwhelmingly accepted thus confirming a central prediction of the theory of rational beliefs: the non-constancy over time of the parameters of the functions which predict returns in each environment. The two-breaks hypothesis is strongly supported by the evidence where the break in 1966:1 receives more statistical support than the break in 1982:1. This is partly due to the small number of observations between 1982:1 and 1992:3. Although the Wald tests establish the fact that the parameters change substantially over time, they must also not exhibit a consistent pattern over time. Table 7 reports the absolute level of the parameters in each environment as specified in (25), (26a), (26b). (Note that in Table 5 the β parameters are *changes* across environments). We indicate in the table the statistical level of significance for testing the hypotheses that the differences of parameter values of consecutive environments are equal to 0[10].

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[10] In fact, in the rest of this section all stars above a parameter will indicate the significance level for testing the equality with 0 of the difference between that parameter and the value of the corresponding parameter in the preceding environment.

Table 5. Parameter estimates of asset pricing model (27)–(28).

| Parameter | One year returns ($J = 4$) | | | | Two year return ($J = 8$) | |
| | Consumption instruments overlap | | Consumption instrument don't overlap | | | |
	Parameter estimates	Standard errors	Parameter estimates	Standard errors	Parameter estimates	Standard errors
Constant	28.3248	(2.6160)	27.4877	(3.0040)	49.4989	(5.9617)
γ	3.0871	(.6417)	2.0166	(.9558)	1.7001	(.6687)
α_2	−.0140	(.0036)	−.0108	(.0041)	−.0202	(.0052)
α_3	.5153	(.1124)	.4180	(.1249)	.2960	(.1264)
α_4	.2691	(.1166)	.2248	(.1335)	.0256	(.1207)
α_5	−.5613	(.2670)	−.2715	(.2934)	−.4430	(.2816)
α_6	−.0927	(.2509)	−.4052	(.2746)	.0570	(.2697)
α_7	−.4872	(.1381)	−.4488	(.1587)	−.3046	(.1590)
β_1^1	−8.9696	(3.7391)	−5.3569	(4.2050)	12.4001	(8.2550)
β_2^1	.0224	(.0055)	.0115	(.0056)	.0223	(.0087)
β_3^1	−.8081	(.1663)	−.8715	(.1898)	−.7337	(.1794)
β_4^1	−.4912	(.1964)	−.4951	(.2245)	−.0129	(.1972)
β_5^1	.2235	(.5017)	.5478	(.5766)	.8941	(.4486)
β_6^1	−.0649	(.4143)	.1069	(.4721)	.6629	(.4840)
β_7^1	.3988	(.1987)	.2414	(.2253)	−.3845	(.2202)
β_1^2	8.3849	(3.5676)	6.9905	(4.1100)	−10.1784	(8.6131)
β_2^2	.0153	(.0080)	.0177	(.0091)	.0285	(.0120)
β_3^2	.3145	(.2536)	.4249	(.2885)	.4141	(.2721)
β_4^2	−.7672	(.2938)	−.7907	(.3355)	−.7196	(.2741)
β_5^2	.9630	(.8141)	.9262	(.9367)	−.4925	(.7938)
β_6^2	1.7927	(.7345)	2.0159	(.8417)	1.3916	(.8204)
β_7^2	−.5317	(.1949)	−.4626	(.2247)	.2133	(.2307)
Adjusted R^2	.6065		.4996		.7340	
Wald test for endogenous uncertainty (p value and χ^2)	p value: .0000 $\chi^2_{20} = 136.13$		p value: .0000 $\chi^2_{20} = 110.15$		p value: .0000 $\chi^2_{20} = 158.79$	

Three environments constitutes much too short a span of time to demonstrate that the parameters have no consistent pattern over time. Yet even the comparison of the three environments demonstrate drastic variability in size and sign which reveal a great deal about the character of the environments and the informational signal of the variables under examination. We discuss each one of them separately.

Table 6. Wald tests for structural change (p values, χ^2 values and d.f.)

Test	One year returns	Two year returns
1966:1 only vs. none	$.0000 \; \chi_7^2 = 76.11$	$.0000 \; \chi_7^2 = 102.30$
1982:1 only vs. none	$.0000 \; \chi_7^2 = 31.58$	$.0000 \; \chi_7^2 = 50.88$
1966:1 and 1982:1 vs. none	$.0000 \; \chi_{14}^2 = 108.26$	$.0000 \; \chi_{14}^2 = 131.96$
1966:1 and 1982:1 vs. 1982:1 only	$.0000 \; \chi_7^2 = 54.48$	$.0000 \; \chi_7^2 = 35.88$
1966:1 and 1982:1 vs. 1966:1 only	$.0172 \; \chi_7^2 = 17.03$	$.0442 \; \chi_7^2 = 14.42$

Table 7. Comparison of the absolute parameter values across environments (one year returns with consumption instruments overlap)

Parameter	1947:1–1965:4	1966:1–1981:4	1982:1–1992:3
Constant	28.3248	19.3552(**)	27.7401(**)
α_2	−.0140	.0084(***)	.0237(*)
α_3	.5153	−.2928(***)	.0217
α_4	.2691	−.2221(**)	−.9893(***)
α_5	−.5613	−.3378	.6252
α_6	−.0927	−.1576	1.6351(**)
α_7	−.4872	−.0884(**)	−.6201(***)

(***) $p < .01$ for significance of difference with parameter of previous environment.
(**) $p < .05$ for significance of difference with parameter of previous environment.
(*) $p < .10$ for significance of difference with parameter of previous environment.

(a) *The signal of commodity prices.* Extracting from Table 7 and computing the corresponding parameter value for the two year returns we have the following picture:

		1947:1–1965:4	1966:1–1981:4	1982:1–1992:3
One year returns	α_3	.5153	−.2928(***)	.0217
	α_4	.2691	−.2221(**)	−.9893(***)
Two year returns	α_3	.2960	−.4377(***)	.0236
	α_4	.0256	.0127	−.7069(***)

These numbers suggest that in the environment of 1947:1–1965:4 when growth rates were above trend, rises in commodity prices preceded periods of expansion of demand and profitability. This is represented by the positive parameters (all three parameters .5153, .2691 and .2960 are significantly different from 0; see Table 5) for

both one year and two year return models. In the environment of 1966:1–1981:4 the situation is reversed: rising commodity prices signaled dangers of inflation, higher interest rates, continuation of the below-trend growth rates and consequently lower profitability. In both the one year and the two year returns the parameters become negative and the differences are statistically significant. In the third environment we find a more complex reaction of asset prices to past changes in commodity prices. Rising commodity prices still signal to some investors lower future returns but with a longer lag. We interpret this longer lag in reaction as representing a tendency of many investors to respond only to *persistent* and sustainable price movements. To see why we offer this interpretation note that after several years of inflation agents learned how to adjust to the direct short term movements in commodity prices. For this reason firms as well as investors were attempting (and partially failing) to forecast the secondary and more complex effects of changes of commodity prices on variables such as monetary policy, corporate balance sheets and long term bond rates. These types of effects operate with a longer lag.

(b) *The signal of industrial production.* The parameters are as follows:

		1947:1–1965:4	1966:1–1981:4	1982:1–1992:3
One year returns	α_5	−.5613	−.3378	.6252
	α_6	−.0927	−.1576	1.6351(**)
Two year returns	α_5	−.4430	.4511(**)	−.0414
	α_6	.0570	.7199	2.1115(**)

In the 1947:1–1965:4 environment, high rates of growth of industrial production signaled to some investors lower returns in the future. This is unambiguous and statistically significant for both one year return (− .5613) as well as two year returns (− .4430) (see Table 5). The difference between environments I and II is more ambiguous: the model with one year returns suggests the same but weaker response in 1966:1–1981:4. However, the analysis of two year returns shows a drastic shift to a *positive* response to observed productivity rises (.4511(**) and .7199 compared with − .4430 and .0570). Unfortunately we cannot estimate these parameters with great precision so that even the economically large change from .0570 to .7199 is not statistically reliable. Yet when we consider all three environments together we note the shift from negative parameters in 1947:1–1965:4 to positive parameters in 1982:1–1992:3. Note particularly the significant shift of the parameters of the *second lag* to large positive values in both models: 1.6351(**) for the model with one year returns and 2.1115(**) for the model with two year returns. Again we interpret these changes to represent the different ways in which productivity increases signaled information (which many investors missed). In 1947:1–1965:4 high rates of growth of industrial production was a good forecast of the business cycle and therefore of *lower* returns to come. The great productivity slow-down of 1966:1–1981:4 changed all of that: sustained productivity rises were taken as strong signals of a recovery from the

long period of low growth rates of output and productivity in manufacturing. This, therefore, became a good signal for *higher* future returns.

(c) *Serial correlation.* As predicted, negative serial correlation is present and the parameter is not constant over time. In the model with one year returns the estimate is $-.4872$ (and significantly different from 0) in environment I and becomes essentially $0(**)$ in environment II. It becomes $-.6201(***)$ in environment III. The pattern is different in the model with two year returns where the estimates are $-.3046$, $-.6891(*)$ and $-.4758$.

We finally comment on the estimation of γ. As seen in Table 6 this parameter is estimated to be 3.09 with one year returns and 1.70 with two year returns. However, employing Hall's [1988] suggestion and utilizing instruments which do not overlap in time with the returns or the lagged returns leads to an estimate of $\gamma = 2.02$. The equation is estimated less precisely where the standard errors are uniformly higher. However, as Table 5 reveals, the parameters in the middle column do not present an economically different picture than the parameters of the left column. The estimate of $\gamma = 2.02$ is within the same economic range of the other estimates and we therefore find no support for Hall's [1988] suggestion.

4 The overvaluation of assets in 1966

In the opening section of this paper we presented the "puzzle" of the declining stock prices; we now return to this puzzle and interpret it in light of the theory developed in this paper. To aid us, we present in Figure 4 two times series:

(a) *Real Dow-Jones.* This is the ratio of the Dow-Jones index and the CPI where the CPI is set at 100 in 1987.
(b) Z_t. This variable is defined by $Z_t = \log Y_t - \hat{\alpha} - \hat{\beta}t$ where Y_t is *real* GNP and $\hat{\alpha}$ and $\hat{\beta}$ are parameters computed by the least squares regression of $\log Y_t$ on t. Z_t is then the detrended logarithm of GNP.

Note that *the slope* of Z_t is exactly the difference between the growth rate of Y_t and the trend $\hat{\beta}$. Although the value of Z_t fluctuated there are two pronounced facts which are central:

(a) For the period *1947–1966* the average growth rate of 4% was *well above-trend*.
(b) For the period *1966–1982* the average growth rate of 2.3% was *well below-trend*.

We turn now to the behavior of investors with Rational Beliefs who traded securities before 1966. With the assistance of the past history of growth environments, all our rational agents knew before 1966 that the economy was experiencing above-trend growth rates. Moreover, if the economy was to continue and experience such high rates of growth, common stocks would be very desirable investments. The theory of Rational Beliefs implies (by (2.2.D)) that at the same time all agents had to believe that with probability 1 the environment of high growth will end at some point. But when would that point be? This was the central problem before 1966. Our theory permits (by (2.2.B)) a random investor to forecast in 1966 that the high growth rates will continue, say, at least until 1974. This optimistic assessment would be allowable even if the event under question was extremely unlikely under the
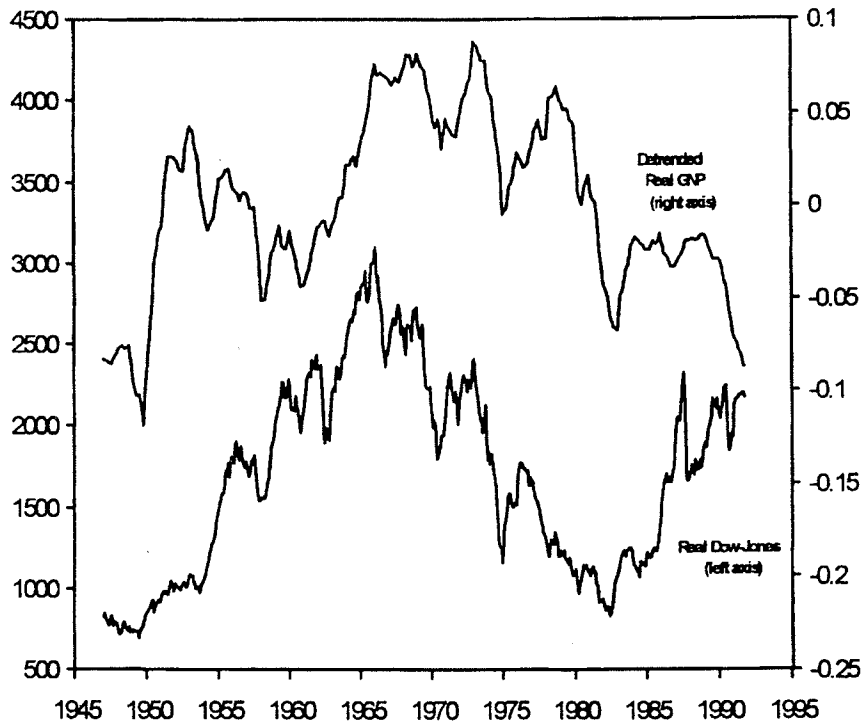
Figure 4

stationary probability that was computed from the empirical frequency distribu-
tions of the lengths of past episodes of above-trend growth rates. An agent could
justify this extreme forecast by noting that although the period of very high growth
rates has been long, it is likely to continue because of the special circumstances of the
1960s (such as the expenditures on the war in Vietnam, the Great Society, etc.). Other
agents may have adopted more conservative beliefs based on the known stationary
probability cited above. We do not know the distribution of forecasts in the market
at that time but what we can assert is that *the distribution of beliefs in 1966 was
heavily tilted in favor of those who forecasted higher prices in the future and continued
rapid, above-trend growth rates for some years to come and certainly past the first
quarter of 1966.*

We can justify the above assertion in two ways. First, one of the central
conclusions of our estimated model in Section (3.3.C) is the presence of structural
change in the stochastic process of returns. The discussion of the Wald tests in
Table 6 shows that the change in 1966:1 was particularly pronounced. As we
explained in detail, *this constitutes a demonstration that the agents did not forecast
correctly the changes.* A simpler argument can be made on the basis of Figure 4.
Those investors who extrapolated the high growth rates beyond 1966 pushed the
Dow-Jones index towards the 1,000 level in the first quarter of 1966 *which is about*

the date where we placed the structural change in the previous section[11]. Putting the argument differently, we are asserting that had the distribution of forecasts in the economy been heavily weighted with agents who believed that the above-trend growth regime would end in 1966, the Dow-Jones would have peaked well before 1966! Needless to say, a rational expectations perspective assuming that agents knew before 1966 that the mean growth rate would change in 1966 was easily rejected by the data.

The argument above highlights a central aspect of the theory of RBE expressed in (2.2.E) about the large number of possible RBE. There is nothing deterministic about the distribution of beliefs in 1966. The fact is that the distribution *could have been different* and this would have resulted in a different RBE and consequently in a different time series of stock prices and returns.

Our central conclusion is that the majority of traders in the market of 1966:1 forecasted continued above-trend growth rates for the U.S. economy and these forecasts were grossly mistaken. After 1966 the U.S. stochastic growth process experienced a major change which we discussed extensively in Section (3.1). We therefore suggest that after 1965:4 the mean value function of the growth rate process declined and in 1965:4 the stock market failed to forecast this important change. *The market of 1966 made a dramatic mistake in the valuation of securities and this excess valuation left a persistent impact on the performance of the securities markets for years to come: it took the market 17 years to digest and adjust for the excess valuation of 1966.* It is clear that this type of empirical implication is central to the theory of RBE (see (3.2.A)–(3.2.B)) and stands in sharp contrast to all variants of the market efficiency theory[12].

What happened after 1966? In as much as the central question prior to 1966 was how long would the high growth era last, the pivotal question after 1966 was how long the slow growth environment would last? Agents who forecasted a short duration of this environment were willing to pay higher prices for securities while traders who forecasted a long duration were the sellers. In this RBE the rate of decline in the real value of securities depended upon the distribution of individual duration forecasts. The rate of decline in the real value of securities was slowed down by an important factor which is also implied by the theory of Rational Beliefs. This factor is the belief of all agents (see (2.2.D)) that with probability 1 the era of slow growth will come to an end at some point and when this occurs, security prices will turn up and go higher. This brings us to the complex issue of price forecasting.

The idea that agents form beliefs about all economic variables *including prices* is central to a RBE and was discussed in detail in Section 2 above. We have explained above that in the context of the post 1966 decline in the real value of securities, agents

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[11] One more above-trend growth spurt took place in the second half of 1967 and this generated a new rally which pushed the Dow-Jones towards 1,000 again in 1968. Thus, the exact quarter of 1966:1 is not essential to our argument.

[12] On the other hand, we note that the theory of RBE does not say that an episode of market decline lasting 17 years is "long" or "short"; the theory analyzes only the properties of such episodes. It is ultimately criteria of economic plausibility that must be employed to determine if 17 years is too long and a 71% decline in value is too step to be satisfactorily viewed just as a random realization of a zero expectations (in 1966) random variable.

believed at each date that with probability 1 prices will turn around and move higher at some early date in the future. An implication of RBE is (see (2.2.A)) that typically there will be some individual predictions of price increases even before there is conclusive evidence for a change in the regime of below-trend growth rates. Our theory therefore implies that an RBE can give rise to erroneous bounded price movements which we have defined earlier as "local bubbles": the expectations of higher (lower) prices which induce actual episodes of rising (falling) prices without being confirmed by later improvements (deterioration) in real economic variables such as growth rates, profits, interest rates, productivity, etc. Our casual inspection of Figure 4 suggests the existence of numerous occasions in the post-War period where the movements of stock prices may be presumed to be local bubbles (positive or negative). A few examples will suffice: in 1962, 1966 and 1987 stock prices fell precipitously while the economy was growing at a rate above or equal to trend and growth rates remained above or equal to trend for at least two years after these dates. The opposite occurred in 1969, 1973, and 1981 when stock prices rose towards the 1,000 Dow-Jones level but for at least two years after the 1,000 level was reached growth rates in the economy were below trend and even negative. The thrust of these observations is to highlight the fact that after 1966 the market was unable to forecast correctly how long the below-trend growth era would last. This is reflected in two facts. First, the long duration it took the market to fall in real value and reach bottom *at the same time in 1982 when the last quarter of below-trend growth rate was being experienced*. Again, the market did not forecast the turning point. Secondly, the belief that below-trend growth should end "soon" induced a sequence of local bubbles, each of which brought prices back to the nominal 1,000 Dow-Jones level. Thus the slow decline from excessive overvaluation is simply a consequence of the inability of the market to forecast correctly the end of the low growth episode.

The issue of trading strategies merits a concluding comment. Even when the slowdown was recognized after 1966, many competing theories were proposed and debated. Two traders on the exchange possessing exactly the same information could be holding opposing beliefs about the duration of the productivity slowdown. Trader A who believed that the slowdown is a phenomenon of long duration would have concluded that there is no likelihood of a strong *and sustained* upward trend in the value of stocks as long as the productivity slowdown lasts. This trader could have adopted a trading strategy of always buying stocks when the Dow-Jones enters, say, the 750 range and sell stocks after the index goes over 950. In fact, this trading strategy was practiced by many investors during that time. Trader B who believed that the slowdown is a short term phenomenon would have adopted a strategy of buying a portfolio of high quality growth stocks and hold it for the long run. He expected the market to resume its upward thrust within a short time and did not want to get caught up in the short term fluctuations. This strategy has been widely employed and represents a very common view.

The fact is that the two traders had the same information but assessed it differently and arrived at opposing trading strategies. The additional fact is that both strategies were compatible with the available information (in the sense that they did not require information which the traders did not have) and were entirely rational. Both strategies were widely employed. However, the ultimate fact is that

Trader A was *right* and Trader B was *wrong*. Trader A consistently made a great deal of money during the 16 years from 1966 to 1982 and Trader B lost a substantial part of this fortune. Could the second trader have noticed that he was losing money while the first trader was making money and that should have convinced him to change his strategy? Perhaps yes but then at some date he had to abandon his conviction that the era of slow growth is just about to end and that up to that date he had been simply unlucky. The theory of RBE permits this kind of disagreement only with respect to phenomena where there is no conclusive evidence for Trader B to change his belief and strategy. Furthermore, after losing a good part of his fortune Trader B found himself vindicated and proved "right" when the market broke 1,000 in the last quarter of 1982 and took off from there. If Trader A was too aggressive and held a short position at that time he would have given back a good part of his winnings.

Appendix

Data sources

The central data bank used in this study is Citibank Economic Database, Citibank N.A. The sources specified here are those provided in the Citibank manual.

1. *Merchandise Imports*: Foreign Transactions in the National Income and Product Accounts—Quarterly, Department of Commerce, Bureau of Economic Analysis (Citibase Series GIMM).
2. *Merchandise Exports*: Foreign Transactions in the National Income and Product Account—Quarterly, Department of Commerce, Bureau of Economic Analysis (Citibase Series GEXM).
3. *Profits of the Manufacturing Sector*: National Income and Product Accounts—Quarterly, Department of Commerce, Bureau of Economic Analysis (Citibase Series GPBMV).
4. *Index of Manufacturing Output (seasonally unadjusted)*: Received from the Federal Reserve.
5. *Index of Manufacturing Output (seasonally adjusted)*: Board of Governors of the Federal Reserve System, Industrial Production Statistical Release G17. (Citibase Series IPMFG).
6. *Consumer Price Index*: U.S. Department of Commerce, Bureau of Economic Analysis (Citibase Series PUNEW).
7. *Price of Common Stocks*: Standard and Poor's Composite Index, Standard and Poor's Corporation, The Outlook. (Citibase Series FSPCOM).
8. *Dividends on Common Stocks*: Computed from the dividend yield series on the Standard and Poor's Composite Index, Standard and Poor's Corporation, The Outlook. (Citibase Series FSDXP).
9. *Short Term Interest Rates*: 3 month Treasury bill rate, Board of Governors of the Federal Reserve System, Selected Interest Rates and Bond Prices, G.13 (415), (Citibase Series FYGN3).
10. *Long Term Bond Rate*: 10 year bond rate, *U.S. Treasury Yield Curves 1926–1988*, by Coleman T.S., L. Fisher and R.G. Ibbotson, Moody's Investor Service. After 1988—Board of Governors of the Federal Reserve System, Selected Interest Rates and Bond Prices, G. 13 (415), (Citibase Series FYGT10).
11. *Price Index of Basic Commodities*: For 1948–1992 used: CRB Commodity Index Report—Raw Materials, Commodity Research Bureau, Inc., One Liberty Place, New York, N.Y., 10006. (Citibase Series PSCMAT). For 1947 interpolated with a regression model on other indexes of basic commodities.

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